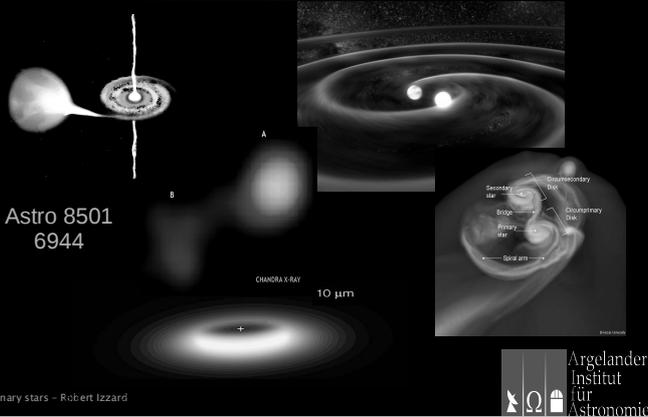
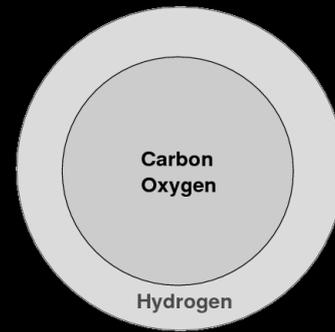


## Binary Stars – Lecture 10



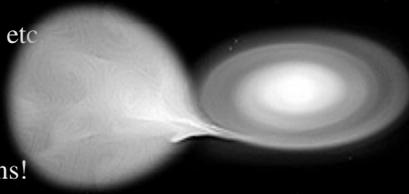
## Classical Nova I



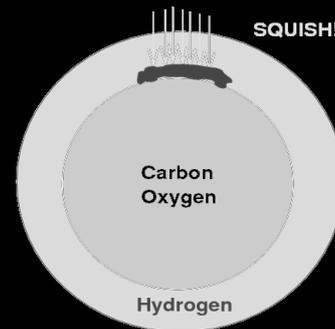
Michael Richmond  
[http://spiff.rit.edu/richmond/asras/sn\\_bh/sn\\_bh.html](http://spiff.rit.edu/richmond/asras/sn_bh/sn_bh.html)

## Cataclysmic Variables

- White Dwarf +
- Low mass star
- WD accreting:
- Disc, outbursts etc
- WDM  $\uparrow$
- Sometimes...  
... Explosions!



## Classical Nova II



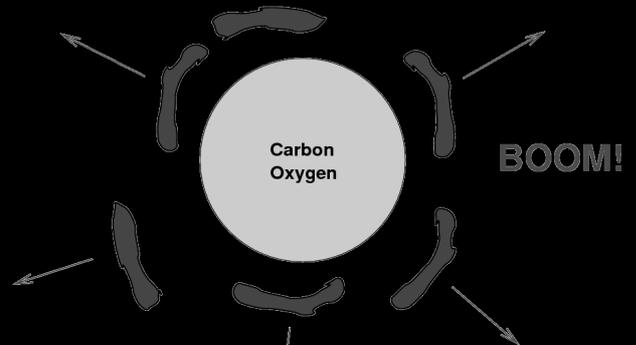
Michael Richmond  
[http://spiff.rit.edu/richmond/asras/sn\\_bh/sn\\_bh.html](http://spiff.rit.edu/richmond/asras/sn_bh/sn_bh.html)

## Accretion Rates onto a WD

$\dot{M} < 10^{-7} M_{\odot} \text{yr}^{-1}$	Thermonuclear Novae
$1.03 < 10^7 \dot{M} < 2.71$	Steady burning
$\dot{M} > 2.7 \times 10^{-7} M_{\odot} \text{yr}^{-1}$	Giant envelope

See e.g. Warner's book (1995)  
Remember the Eddington limit!

## Classical Nova III



Michael Richmond  
[http://spiff.rit.edu/richmond/asras/sn\\_bh/sn\\_bh.html](http://spiff.rit.edu/richmond/asras/sn_bh/sn_bh.html)

# Thermonuclear Nova Properties

- Galactic rate  $\sim 35 \pm 11 \text{ yr}^{-1}$  ( $\sim 5$  observed)
- Mass return  $\sim 4 \times 10^{-4} M_{\odot}$  in 100 – 1000 s
- Energy  $E \sim 10^{45}$  erg
- Luminosity  $L \sim 10^{4-5} L_{\odot}$  (c.f.  $10^{10} L_{\odot}$  for SNe)
- Peak  $T \sim 0.1 - 0.4 \text{ GK}$
- Ejection velocity  $\sim 10^3 \text{ km s}^{-1}$  (c.f.  $\sim 10^4$  for SNe)
- Binary progenitor  $P \sim 1 - 12$  hours CVs!
- Periodic: typically  $10^4 - 10^5$  years
- Rise time  $\sim 1 - 2$  days

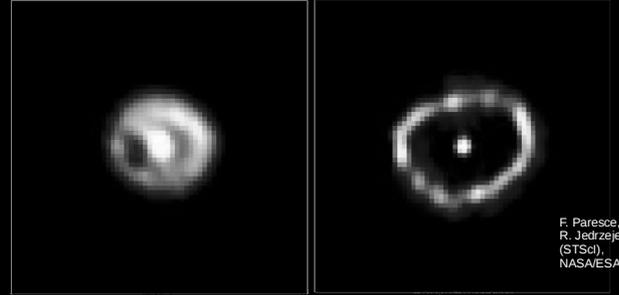


Binary stars – Robert Izzard



# Nova Cygni 1992

Hubble Space Telescope  
Faint Object Camera



Pre-COSTAR  
Raw Image

With COSTAR  
Raw Image

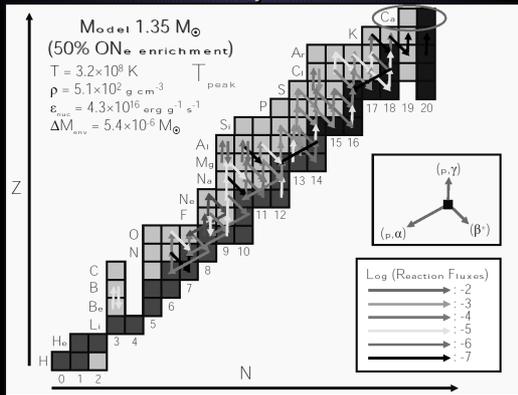
Note the "bar" in the orbital plane

Binary stars – Robert Izzard



F. Paresce,  
R. Jedrzelewski  
(STScI),  
NASA/ESA

# Nucleosynthesis

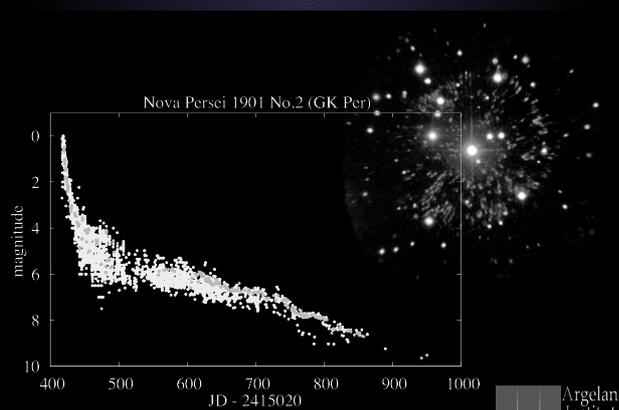


Stolen from Jordi Jose

Binary stars – Robert Izzard



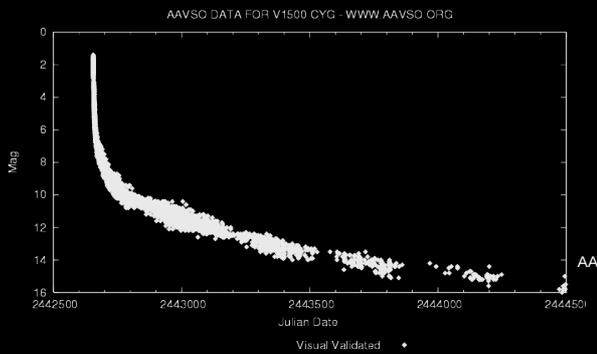
# GK Per



Binary stars – Robert Izzard



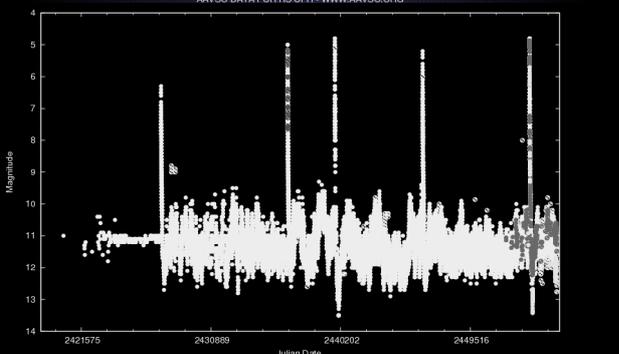
# V1500 Cygni



Binary stars – Robert Izzard



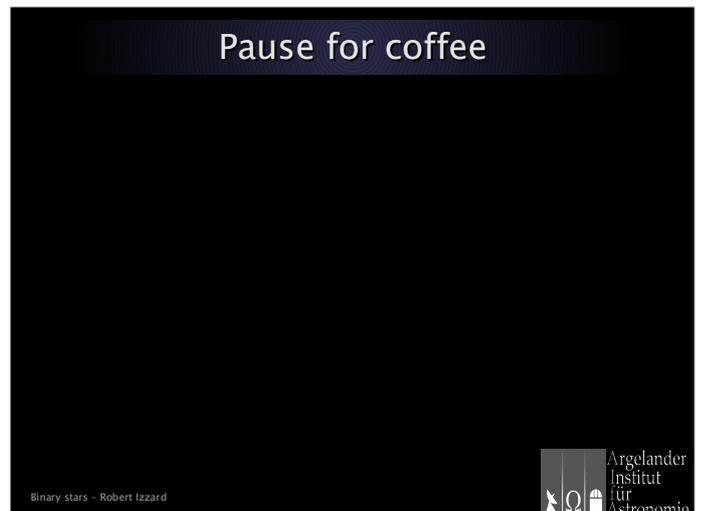
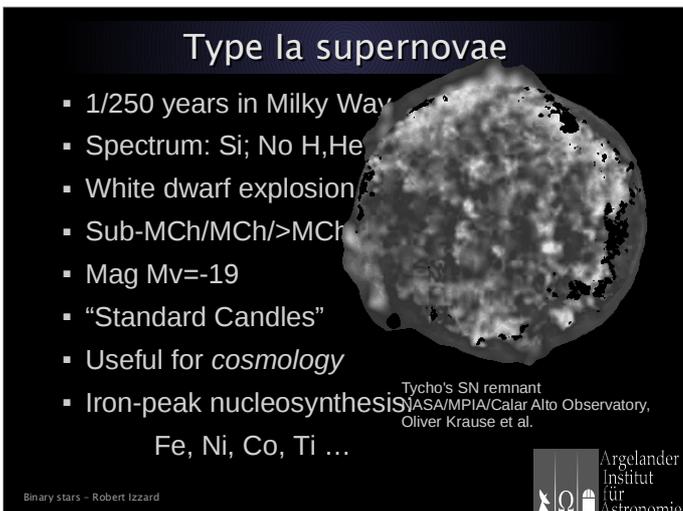
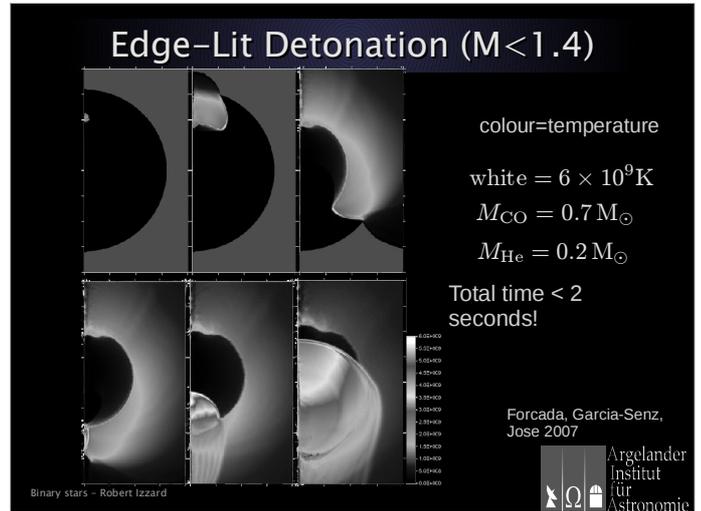
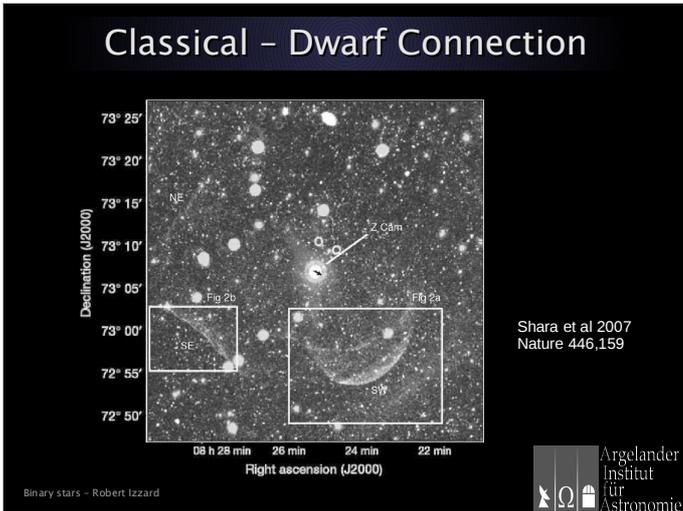
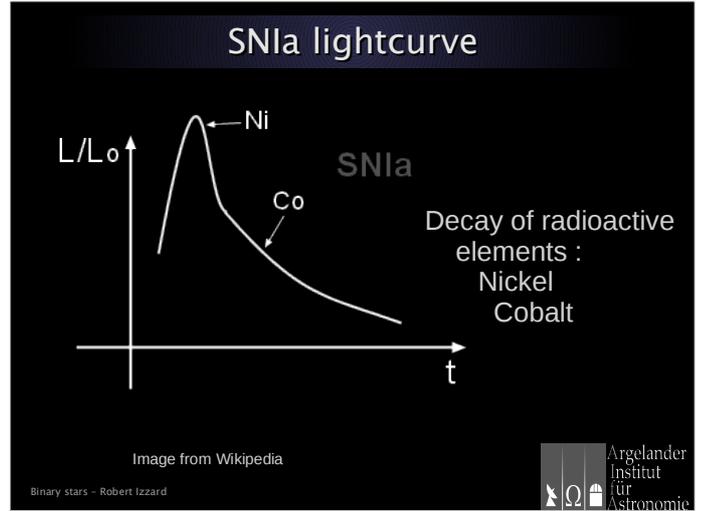
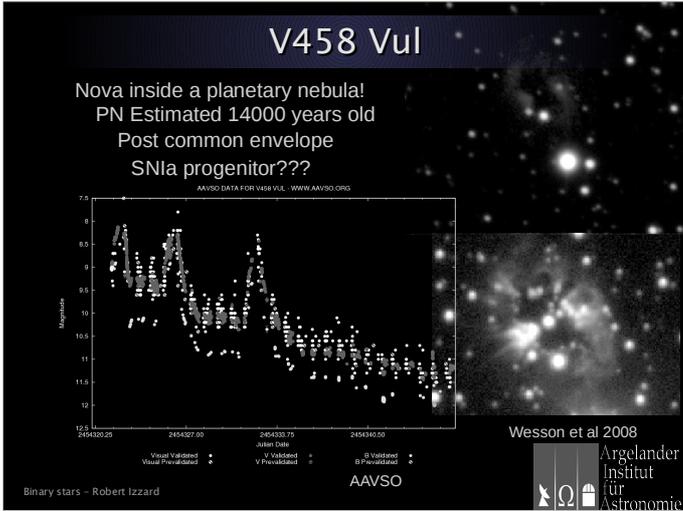
# RS Ophiuchi



20 year period (1898, 1933, 1958, 1967, 1985, 2006)

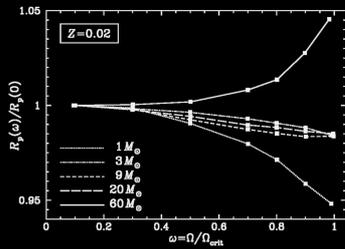
Binary stars – Robert Izzard





### An aside: dimensions of rotating stars

- Can we treat stars as essentially *single stars*?
- Polar radius is approx const.



Variations in the polar radius as a function of the ratio  $\omega = \Omega/\Omega_{\text{crit}}$ , normalized to the non-rotating value, for various masses at standard metallicity.

Ekstrom et al  
2008 A&A  
478, 467

### Fitting Formulae

- Time evolution function of  $\tau = t/t_{\text{MS}}$

$$t_{\text{MS}} = \frac{2550 + 669M^{2.5} + M^{4.5}}{0.0327M^{1.5} + 0.346M^{4.5}}$$

- Then

$$\log_{10} L = \log_{10} L_0 + \alpha \tau_{\text{MS}} + \beta \tau_{\text{MS}}^2$$

$$\log_{10} R = \log_{10} R_0 + \alpha' \tau_{\text{MS}} + \beta' \tau_{\text{MS}} + \gamma' \tau_{\text{MS}}^3$$

### Rapid Stellar Models

- Creating *detailed* stellar models is slow and difficult
- Rapid or synthetic stellar models are faster
- Replace details solver with pre-solved model set:

Fitting formulae

Or lookup tables

- Sacrifice (usually unwanted) details for speed: up to 10,000,000 times faster.



### Fitting formulae

$$\alpha = \begin{cases} 0.2594 + 0.1348 \log_{10} M & M \leq 1.334 \\ 0.09209 + 0.05934 \log_{10} M & M > 1.334 \end{cases}$$

$$\beta = \begin{cases} 0.144 - 0.833 \log_{10} M & M \leq 1.334 \\ 0.3756 \log_{10} M - 0.1744 (\log_{10} M)^2 & M > 1.334 \end{cases}$$

$$\alpha' = \begin{cases} 0 & M \leq 1.334 \\ 0.1509 + 0.1709 \log_{10} M & M > 1.334 \end{cases}$$

$$\beta' = \begin{cases} 0.2226 \log_{10} M & M \leq 1.334 \\ -0.4805 \log_{10} M & M > 1.334 \end{cases}$$

$$\gamma' = \begin{cases} 0.1151 & M \leq 1.334 \\ 0.5083 \log_{10} M & M > 1.334 \end{cases}$$

Even more complicated formulae apply for later phases of evolution!  
But computers *do not care* ...

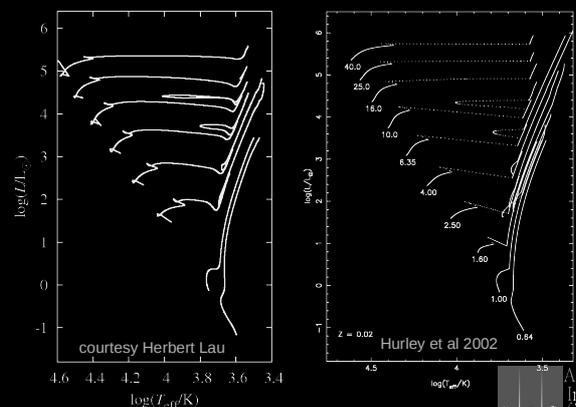
### Fitting Formulae

- Eggleton, Fitchett, Tout 1989, Hurley et al 2000, 2002
- Zero-age main sequence:

$$L_0 = \begin{cases} \frac{1.107M^3 + 240.7M^9}{1 + 281.9M^4} & M \leq 1.093 \\ \frac{13990M^5}{M^4 + 2151M^2 + 3908M + 9536} & M \geq 1.093 \end{cases}$$

$$R_0 = \begin{cases} \frac{0.1148M^{1.25} + 0.8604M^{3.25}}{0.04651 + M^2} & M \leq 1.334 \\ \frac{1.968M^{2.887} - 0.7388M^{1.679}}{1.821M^{2.337} - 1} & M \geq 1.334 \end{cases}$$

### Real vs Synthetic HRD



## Pros and Cons

- Pros

Faster to compute  
Stable

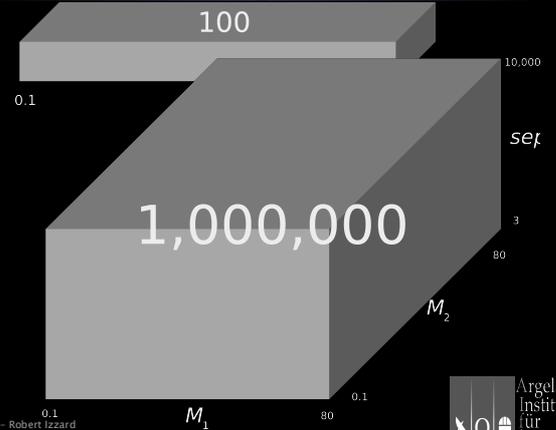
- Cons

Fixed input physics (but could use tables!)  
Discard of potentially useful information  
Off-grid treatment  
Fitting errors (<5%)

$$\log_{10} L = \log_{10} L_0 + \alpha \tau_{MS} + \beta \tau_{MS}^2$$

$$\log_{10} R = \log_{10} R_0 + \alpha' \tau_{MS} + \beta' \tau_{MS} + \gamma \tau_{MS}^3$$

## Parameter Spaces

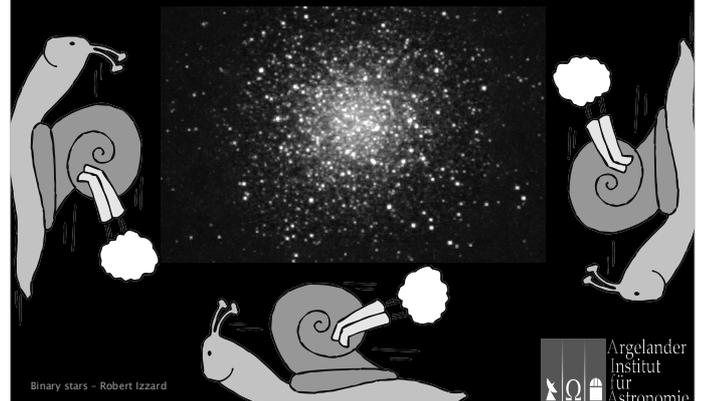


## Population Synthesis

*The process of combining stellar models into a stellar population upon which meaningful statistical analysis can be performed and compared to observations to better constrain the underlying physics.*

1. Make your stellar models
2. Weight these according to mass, separation, time etc.
3. Extract simulated value(s)-compare
4. Determine the "real-life" distribution from obs.
5. Compare the two, see what's wrong
6. Refine your stellar models
7. Return to step 1 until you are happy  
(or funding runs out)

## Popsyn + rapid code



## The Parameter Space Problem

- To make a single star population, one parameter  $M_1$   
only: Mass  $\sim N \times \Delta t$
- Runtime is
- Binaries many parameters :  
Primary mass  $M_1$   
Secondary mass  $M_2$   
Sep/Period  $a$  or  $P$   
Maybe more e.g.  $e$   
 $\sim N^3 \times \Delta t$
- Runtime

## Discretising Parameter Space

- Single Stars

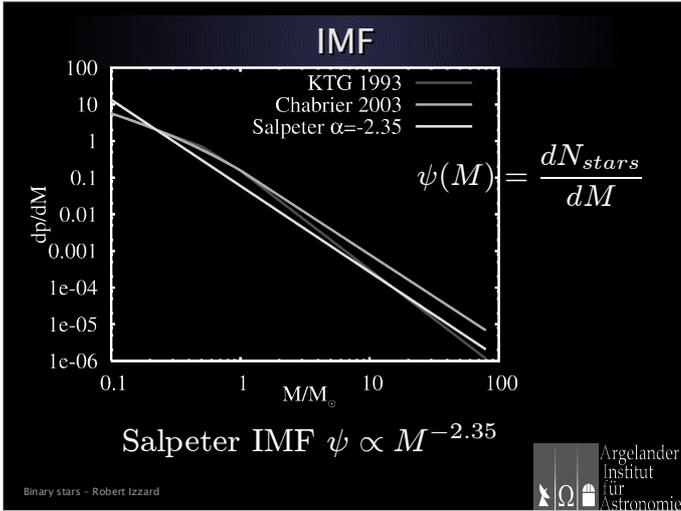
$$\delta \ln M = \frac{\ln M_{\max} - \ln M_{\min}}{n}$$

- Each star has a probability of existence

$$\delta p_i = \psi(M_i) \delta \ln M$$

- Where  $\psi$  is the initial mass function

$$\sum_i \delta p_i = 1$$



### Stellar accounts

- Define
  - $\delta(\text{phase}) = 1$  during the phase,
  - $= 0$  otherwise.
- Time a star spends in a phase of interest
 
$$\Delta t_i = \sum_{t=t_{\min}}^{t_{\max}} \delta(\text{phase at } t)_i \delta t$$

Argelander Institut für Astronomie

### Discretising Parameter Space

- Binary Stars
 
$$\delta \ln x = \frac{\ln x_{\max} - \ln x_{\min}}{n_x}$$

where  $x$  is  $M_1, M_2, a, P, e, \dots$
- Each star has a probability of existence
 
$$\delta p_i = \Psi_i(M_1, M_2, a) \delta V$$
- Where  $\Psi$  is the *initial distribution function*

Argelander Institut für Astronomie

### Stellar accounts

- The number of stars in the phase is
 
$$\text{count} = \sum_i S \delta p_i \Delta t_i$$

$$= \sum_i S \delta p_i \sum_{t_{\min}}^{t_{\max}} \delta(\text{phase})_i \delta t$$

where  $S$  is the star formation rate
- In general we have to convolve a birth function with a star formation rate function

Argelander Institut für Astronomie

### Initial Distribution Function

$$\Psi_i = \psi(M_{1i}) \phi(M_{2i}/M_{1i}) \chi(a_i)$$

$$\psi(M_1) = \psi(M)$$

$$\phi\left(q = \frac{M_1}{M_2}\right) = \text{constant}$$

$$\chi(a) \propto a^{-1}$$

$$\chi(\ln a) = \text{constant.}$$

$$\delta p_i = \Psi_i \delta V_i$$

$$\delta V = \delta \ln M_1 \delta \ln M_2 \delta \ln a$$

$$\sum_i \delta p_i = 1$$

Argelander Institut für Astronomie

### Stellar accounts

- Simple case :  $S = \text{constant}$
- Divide counts to get ratios :  $S$  drops out
 
$$\text{ratio} = \frac{[\sum_i \delta p_i \Delta t_i]_1}{[\sum_i \delta p_i \Delta t_i]_2}$$

Galactic SFR  
Chiappini et al 1997

Argelander Institut für Astronomie

## Stellar accounts

- The number of stars in the phase is

$$\sum_i S \delta p_i \Delta t_i$$

where  $S$  is the star formation rate

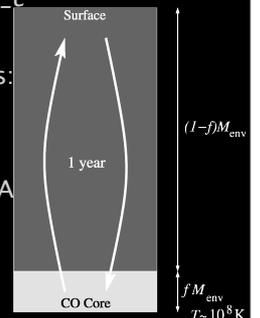
- In general we have to convolve a birth function with a star formation rate function

$$\sum_{t'_{\min}}^{t'_{\max}} \sum_i S(t) \delta p_i \delta(\text{phase at } t')_i \delta t'$$

Binary stars - Robert Izzard

## binary\_c/nucsyn

- Added nucleosynthesis to binary\_c
- First and second dredge up
- TPAGB based on Karakas' models:
  - Third dredge up
  - Hot-bottom burning (CNO, NeNa, MgA)
  - S-process (Torino group)
- SN II/Ibc yields, novae
- Thermohaline mixing
- Physics updates over last few years



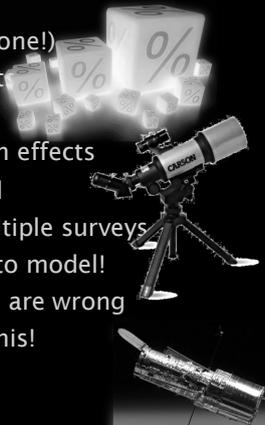
Binary stars - Robert Izzard

## Compare to Observations

- Statistics!

Boring (but not for everyone!)  
Necessary e.g.  $\chi^2$ , KS test  
Key to good science

- Beware observational selection effects
  - Often very hard to model
  - Data combined from multiple surveys might be impossible to model!
  - Sometimes whole papers are wrong because they neglect this! (not deliberately)



Binary stars - Robert Izzard

## Some examples of binary\_c

- Remember to try it yourself!
- <http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cg>

Cluster Name	Star 1 Mass (M <sub>☉</sub> )	Star 2 Mass (M <sub>☉</sub> )	Star 1 Type	Star 2 Type	Separation (R <sub>☉</sub> )	Age (Myr)	Star 1 (M <sub>☉</sub> )	Star 2 (M <sub>☉</sub> )	Notes	Image
0.0000	14.000	0.000	Main Sequence	Main Sequence	100.000	25.92	0.000	0.106	In the beginning there was a star.	
14.0000	12.718	0.000	Reddwarfing Cap	Main Sequence	101.384	26.61	0.000	0.266	Star Type Change	
14.1460	12.718	0.000	Reddwarfing Cap	Main Sequence	101.384	26.64	0.000	0.100	Begin Roche Lobe Overflow	
14.1460	12.718	0.000	Reddwarfing Cap	Main Sequence	101.384	26.64	0.000	0.100	Common Envelope Evolution in	
14.1460	12.449	0.000	Main Sequence	Main Sequence	12.748	1.72	0.000	0.006	Common Envelope Evolution	
14.1460	12.449	0.000	Main Sequence	Main Sequence	12.748	1.72	0.000	0.112	End of Roche Lobe Overflow	
14.1728	12.842	0.014	Reddwarfing Cap	Reddwarfing Cap	13.209	1.88	0.000	0.103	Star Type Change	
14.2012	2.078	0.023	Reddwarfing Cap	Main Sequence	13.209	1.89	0.000	0.003	Begin Roche Lobe Overflow	

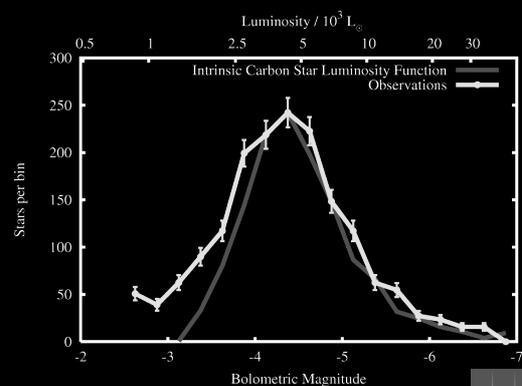
Binary stars

## A rapid code: binary\_c

- My code, my lectures, so ...
- Based on SSE/BSE of EFT89, Hurley et al 2000, 2002 (e.g. see prev. eqs)
- Has fitting functions for stellar evolution
- +orbital algorithm: RLOF, Wind, Tides
- Common env., Novae, SNe Ia, Mergers etc.
- Online
- <http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cgi>

Binary stars - Robert Izzard

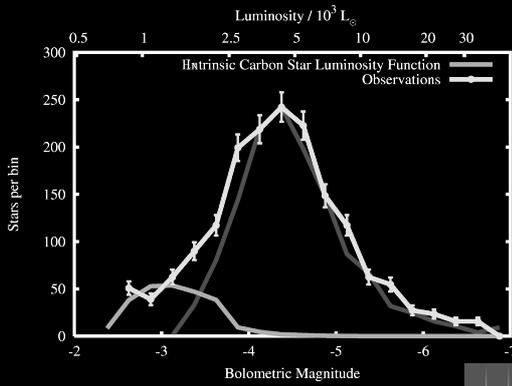
## Low-L Carbon Stars



Binary stars - Robert Izzard

Izzard and Tout 2004

## Low-L Carbon Stars

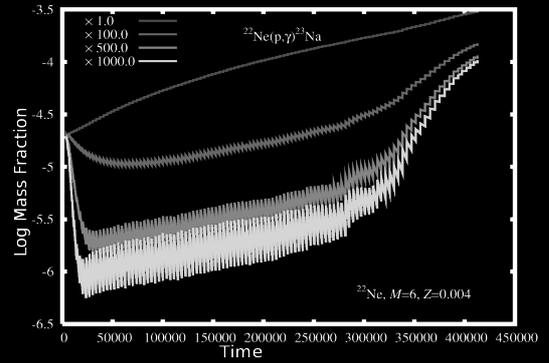


Binary stars - Robert Izzard

Izzard and Tout 2004



## Nuclear Burning Rates

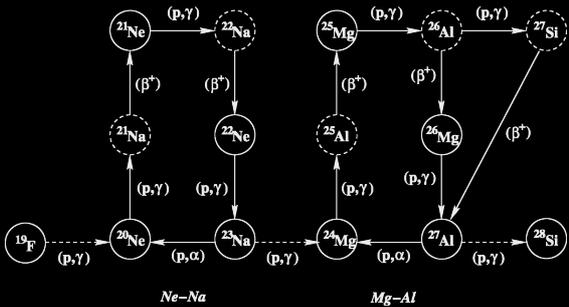


Binary stars - Robert Izzard

Izzard et al. 2007



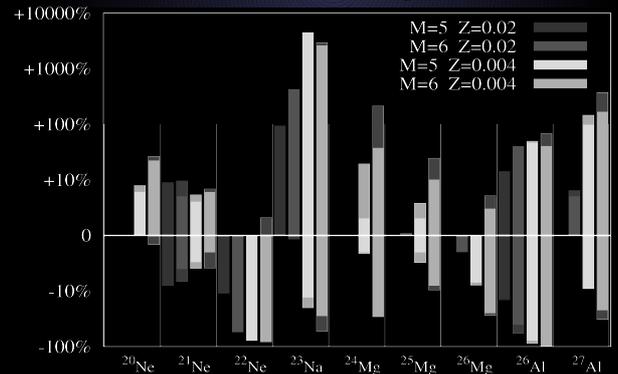
## Nuclear Burning Rates



Binary stars - Robert Izzard



## Nuclear Burning Rates



Binary stars - Robert Izzard

Izzard et al. 2007



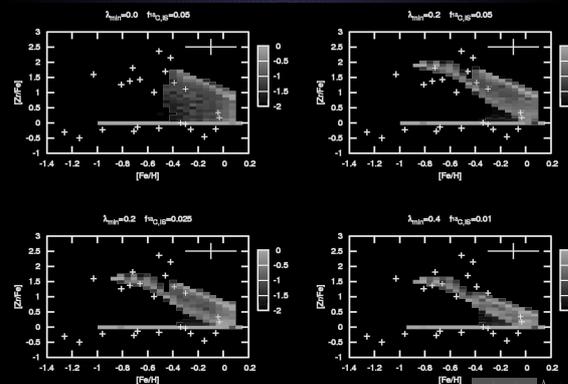
## Nuclear Burning Rates

Rate			Source
$^{20}\text{Ne}(p, \gamma)^{21}\text{Na}(\beta^+)^{21}\text{Ne}$	-50%	+50%	NACRE
$^{21}\text{Ne}(p, \gamma)^{22}\text{Na}(\beta^+)^{22}\text{Ne}$	-20%	+20%	Iliadis et al. 2001
$^{22}\text{Ne}(p, \gamma)^{23}\text{Na}$	-50%	$\times 2000$	Hale et al. 2001
$^{23}\text{Na}(p, \alpha)^{20}\text{Ne}$	-30%	+30%	Rowland et al. 2004
$^{23}\text{Na}(p, \gamma)^{24}\text{Mg}$	/40	$\times 10$	Rowland et al. 2004
$^{24}\text{Mg}(p, \gamma)^{25}\text{Al}(\beta^+)^{25}\text{Mg}$	-17%	+20%	Powell et al. 1999
$^{25}\text{Mg}(p, \gamma)^{26}\text{Al}(\beta^+)^{26}\text{Mg}$	-50%	$\times 1.5$	Iliadis et al. 2001
$^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$	/4	$\times 10$	Iliadis et al. 2001
$^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$	-25%	$\times 3$	Iliadis et al. 2001
$^{26}\text{Al}(p, \gamma)^{27}\text{Si}$	/2	$\times 600$	Iliadis et al. 2001

Binary stars - Robert Izzard



## s-process in post-AGB

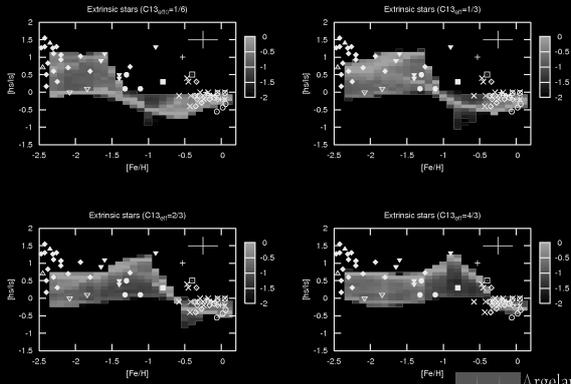


Binary stars - Robert Izzard

Bonacic et al. 2007



# s-process in post-AGB

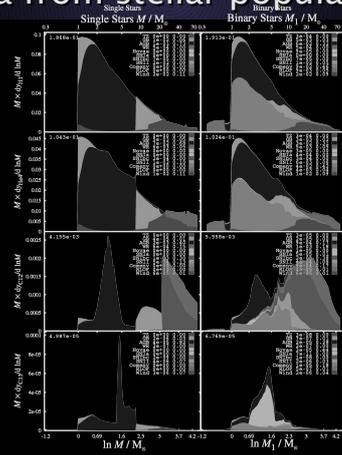


Bonacic et al. 2007



Binary stars – Robert Izzard

# Ejecta from stellar populations

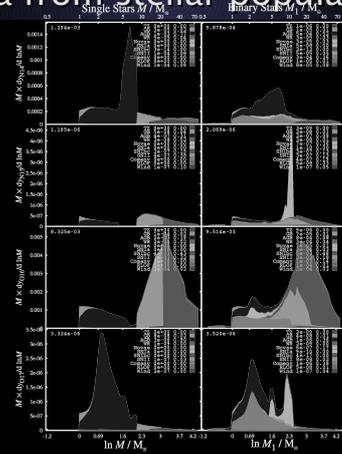


Izzard PhD!

Binary stars – Robert Izzard



# Ejecta from stellar populations



Izzard PhD!

Binary stars – Robert Izzard

