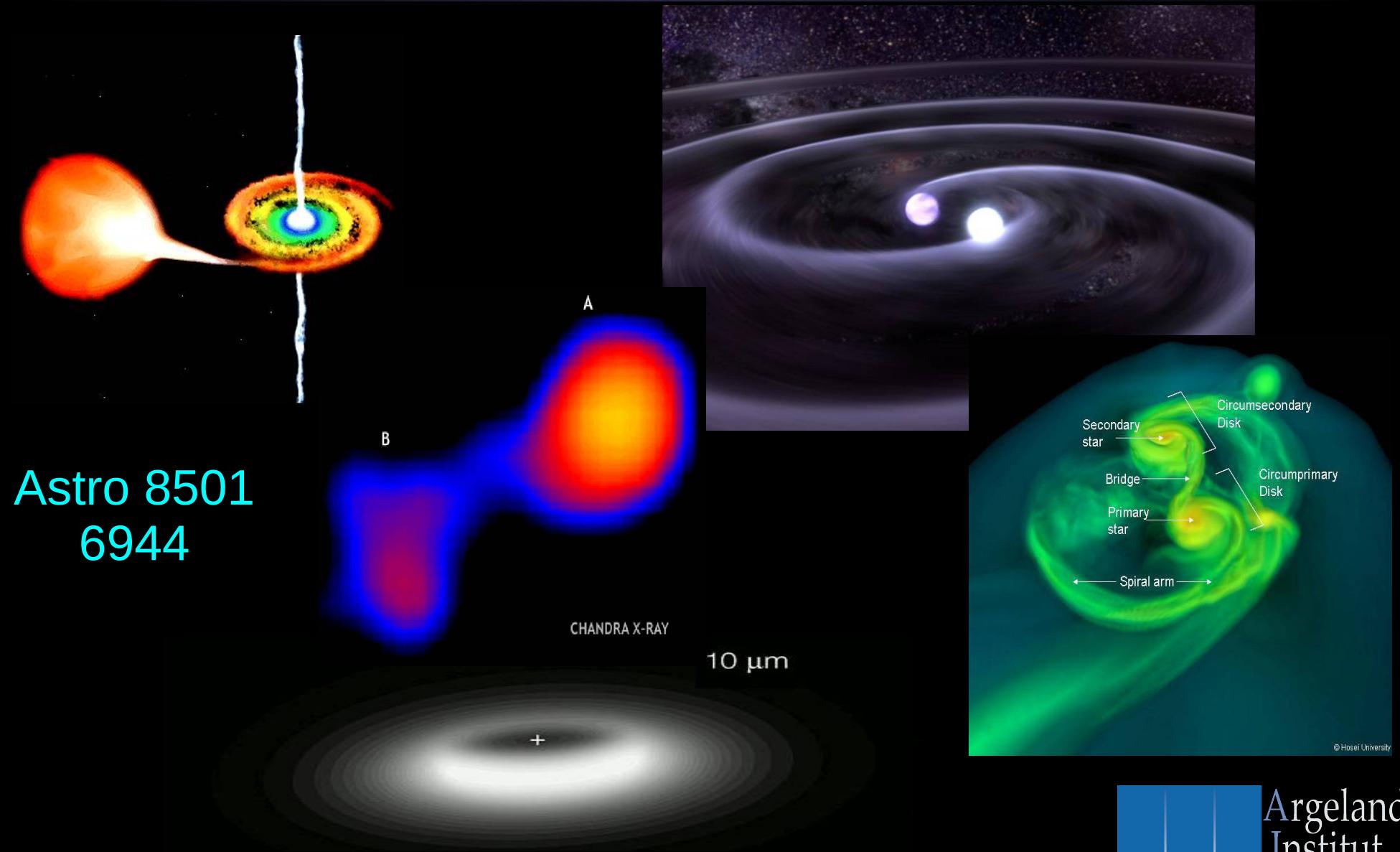
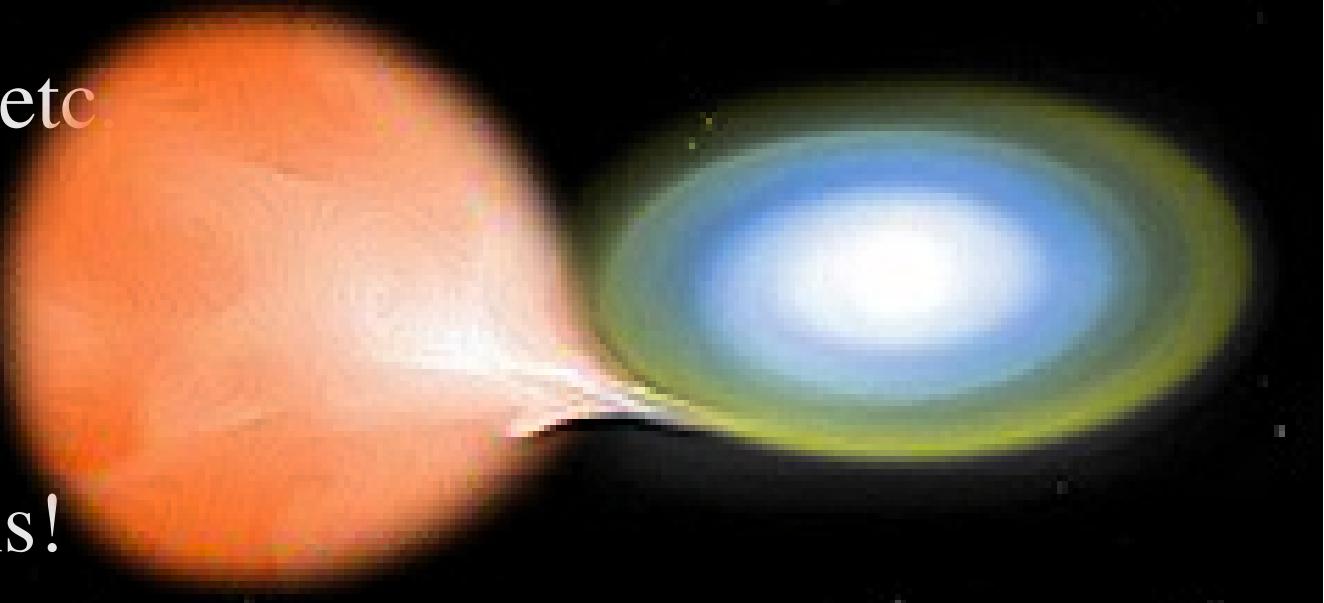


# Binary Stars – Lecture 10



# Cataclysmic Variables

- White Dwarf +
- Low mass star
- WD accreting:
- Disc, outbursts etc
- $WDM \uparrow$
- Sometimes...  
... Explosions!

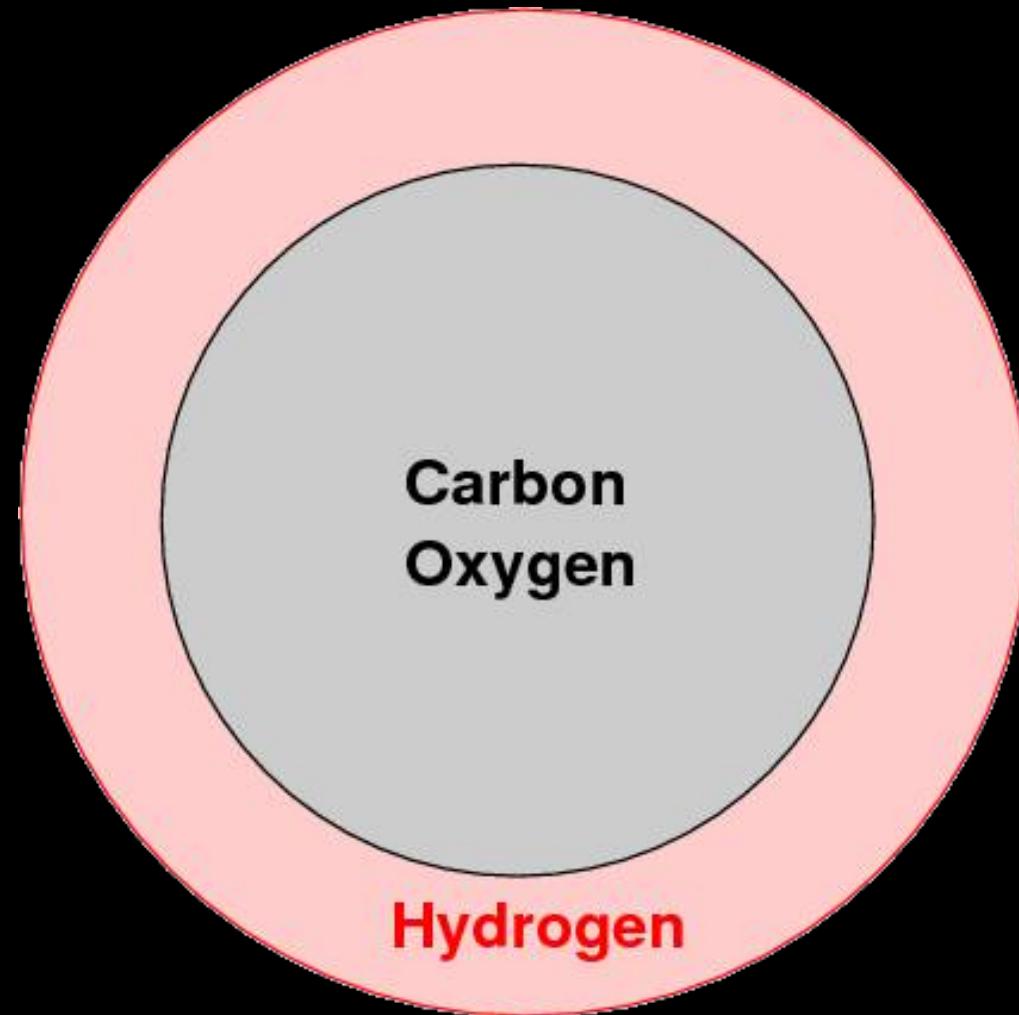


# Accretion Rates onto a WD

$\dot{M} < 10^{-7} M_{\odot} \text{ yr}^{-1}$	Thermonuclear Novae
$1.03 < 10^7 \dot{M} < 2.71$	Steady burning
$\dot{M} > 2.7 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$	Giant envelope

See e.g. Warner's book (1995)  
Remember the Eddington limit!

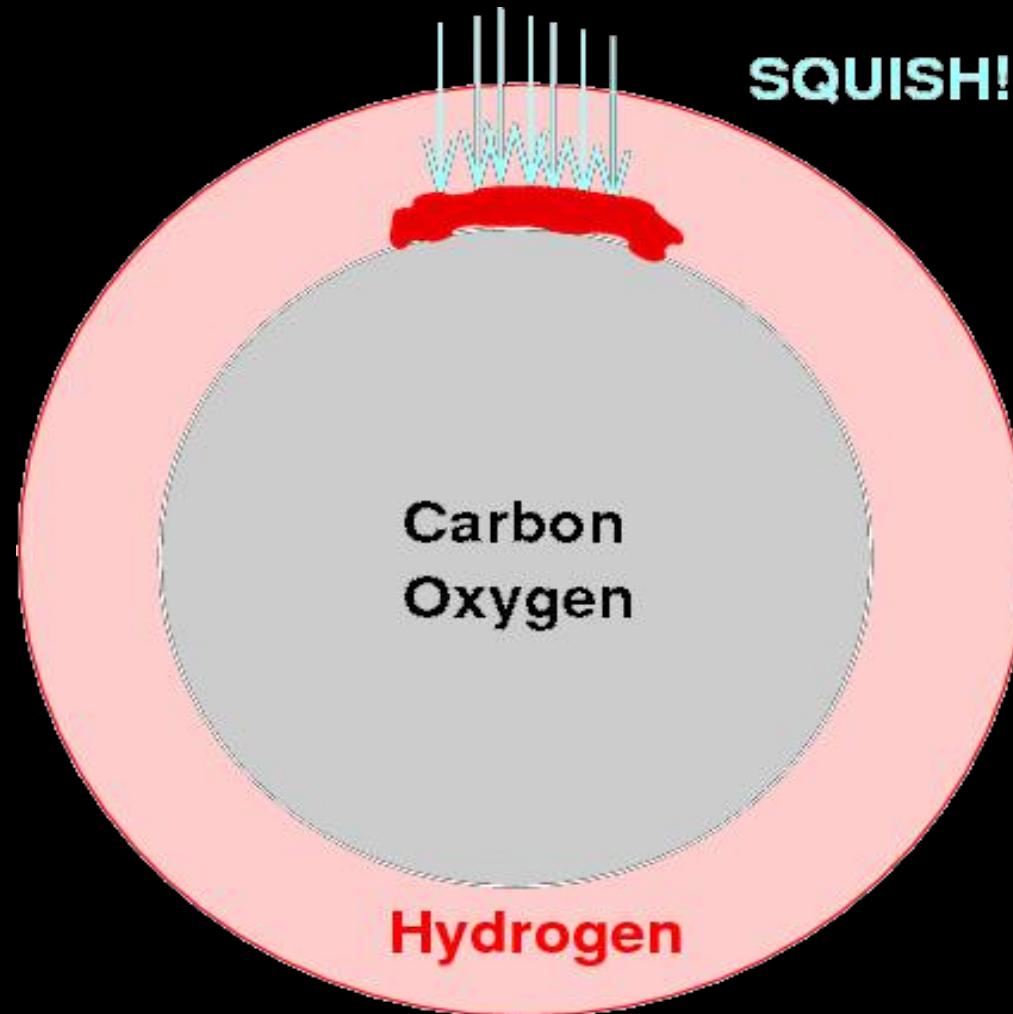
# Classical Nova I



Michael Richmond

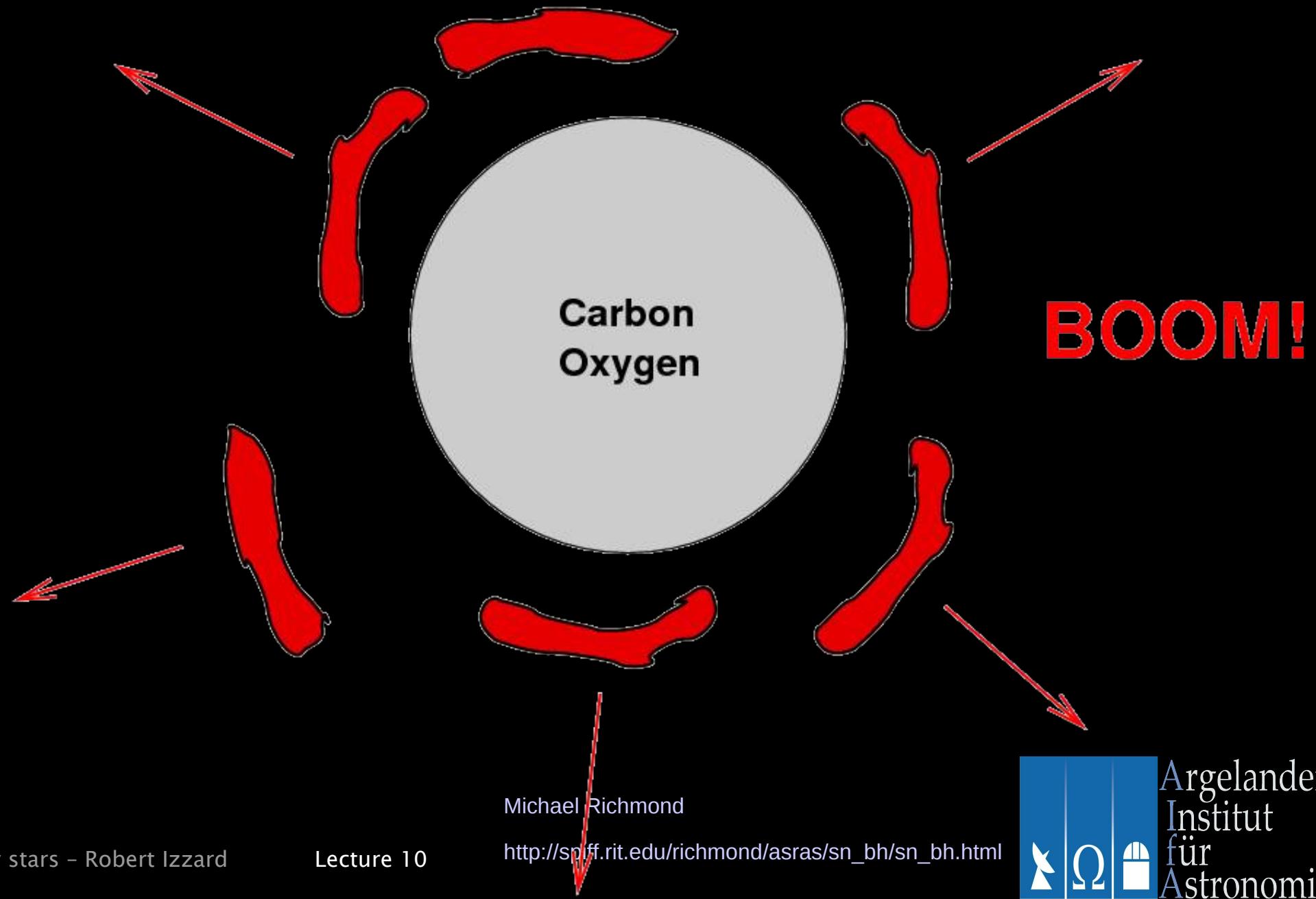
[http://spiff.rit.edu/richmond/asras/sn\\_bh/sn\\_bh.html](http://spiff.rit.edu/richmond/asras/sn_bh/sn_bh.html)

# Classical Nova II



Michael Richmond

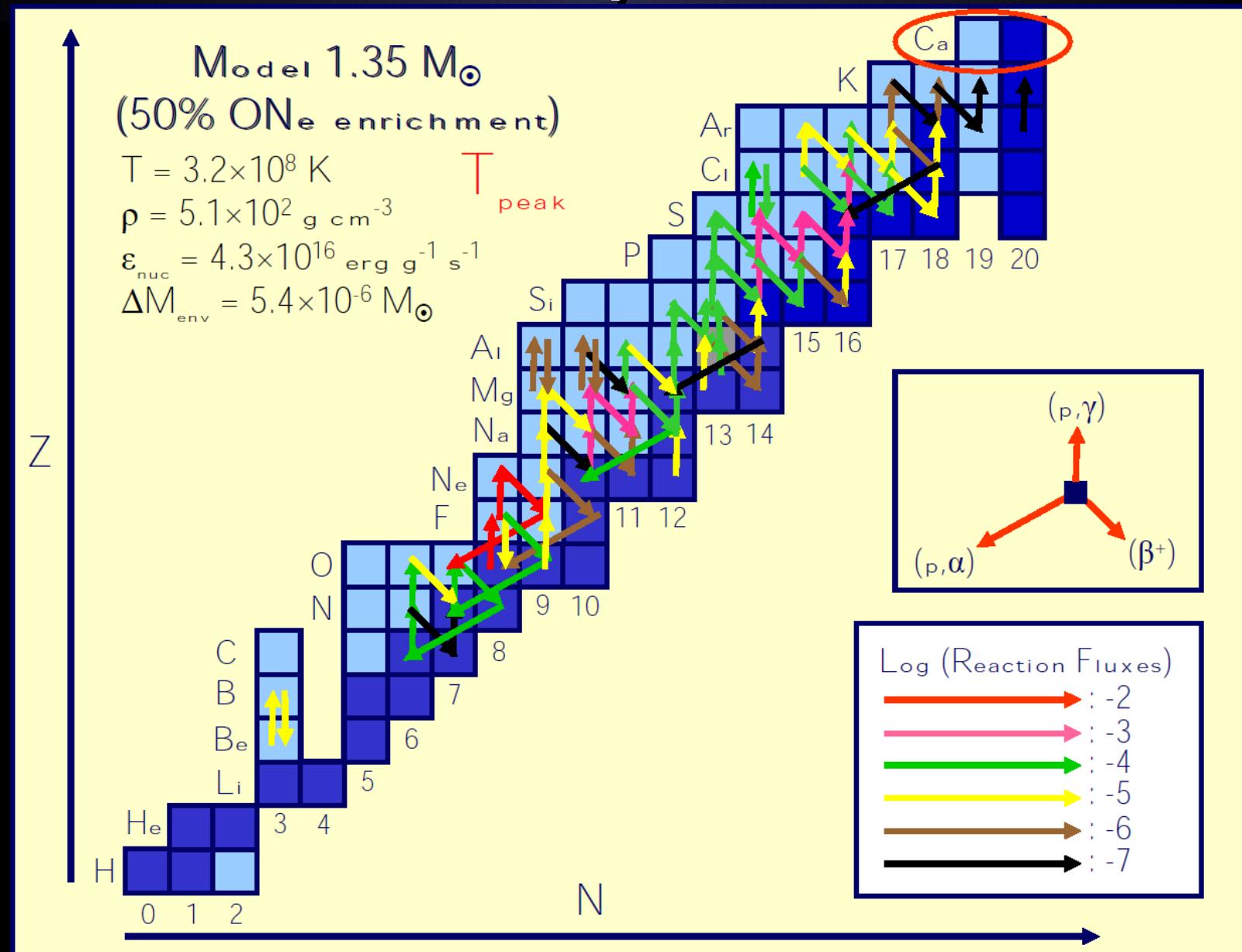
# Classical Nova III



# Thermonuclear Nova Properties

- Galactic rate  $\sim 35 \pm 11 \text{ yr}^{-1}$  ( $\sim 5$  observed)
- Mass return  $\sim 4 \times 10^{-4} M_{\odot}$  in  $100 - 1000$  s
- Energy  $E \sim 10^{45}$  erg
- Luminosity  $L \sim 10^{4-5} L_{\odot}$  (c.f.  $10^{10} L_{\odot}$  for SNe)
- Peak  $T \sim 0.1 - 0.4$  GK
- Ejection velocity  $\sim 10^3 \text{ km s}^{-1}$  (c.f.  $\sim 10^4$  for SNe)
- Binary progenitors  $P \sim 1 - 12$  hours  
CVs!
- Periodic: typically  $10^4 - 10^5$  years
- Rise time  $\sim 1 - 2$  days

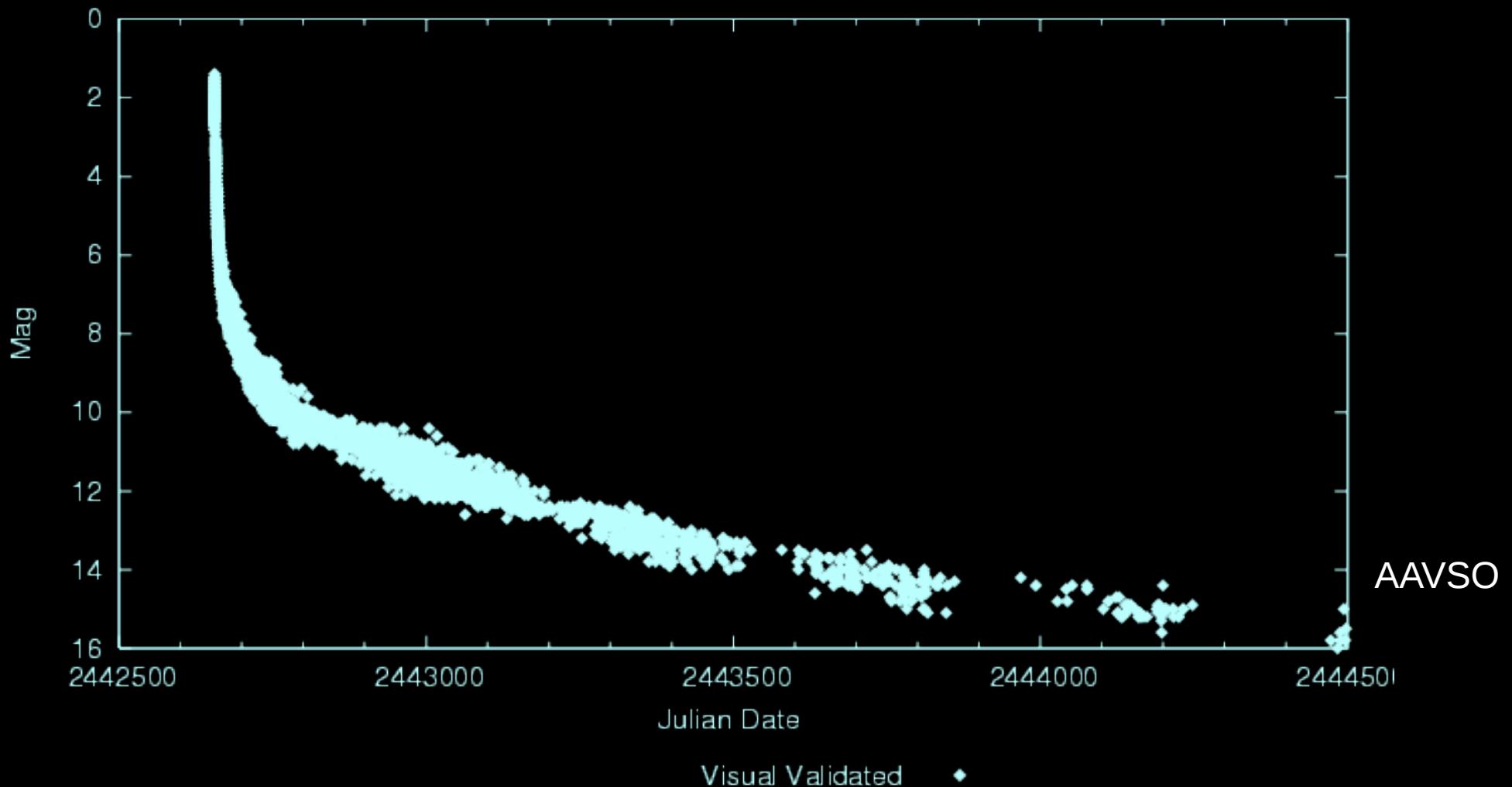
# Nucleosynthesis



Stolen from Jordi Jose

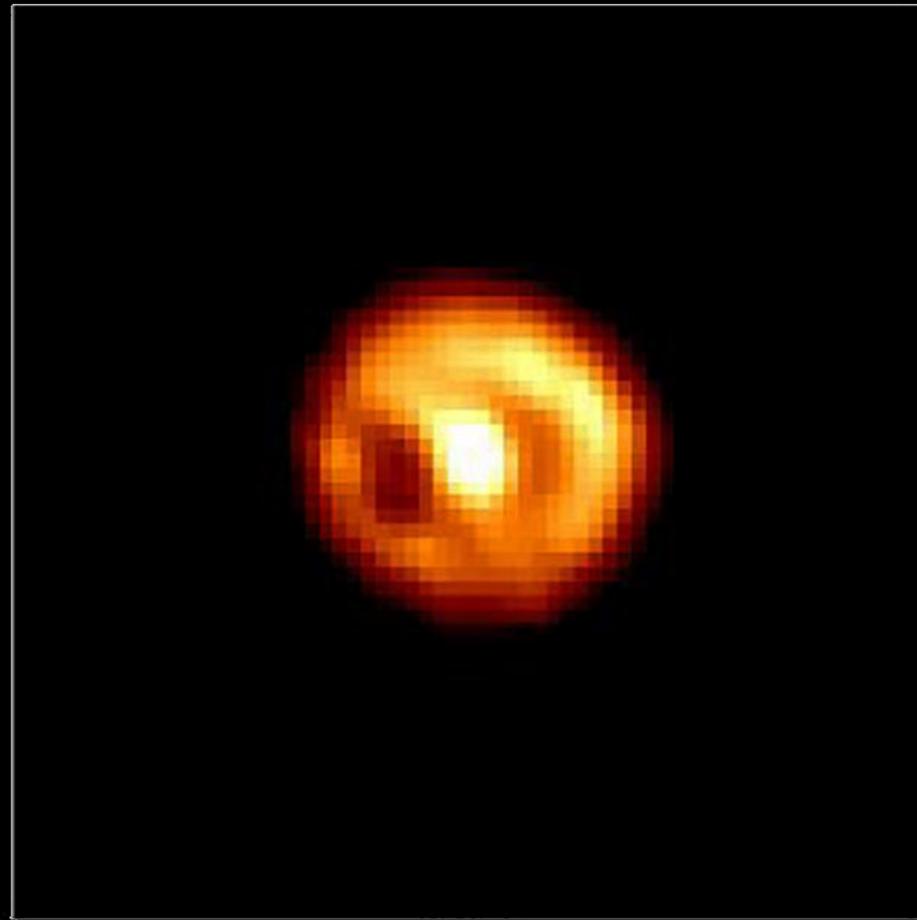
# V1 500 Cygni

AAVSO DATA FOR V1500 CYG - WWW.AAVSO.ORG



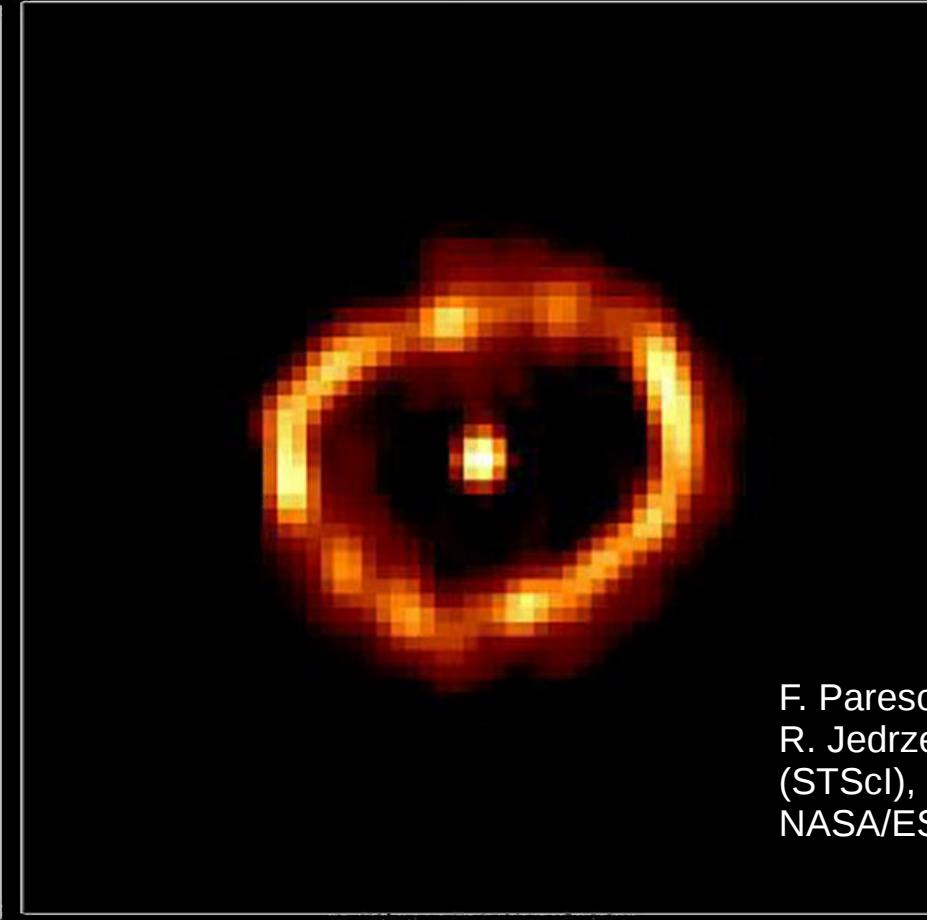
# Nova Cygni 1992

Hubble Space Telescope  
Faint Object Camera



Pre-COSTAR  
Raw Image

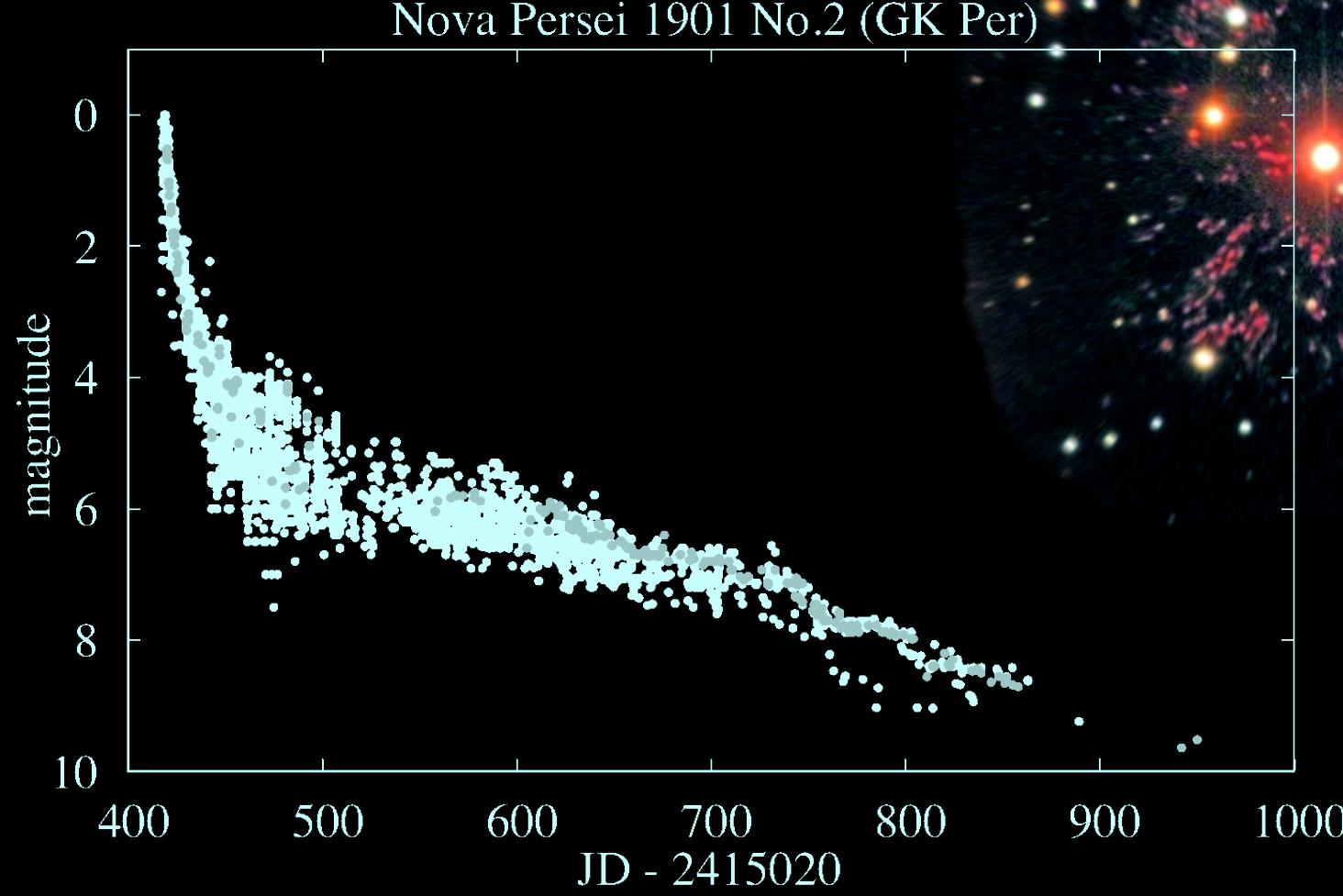
Note the “bar” in the orbital plane



With COSTAR  
Raw Image

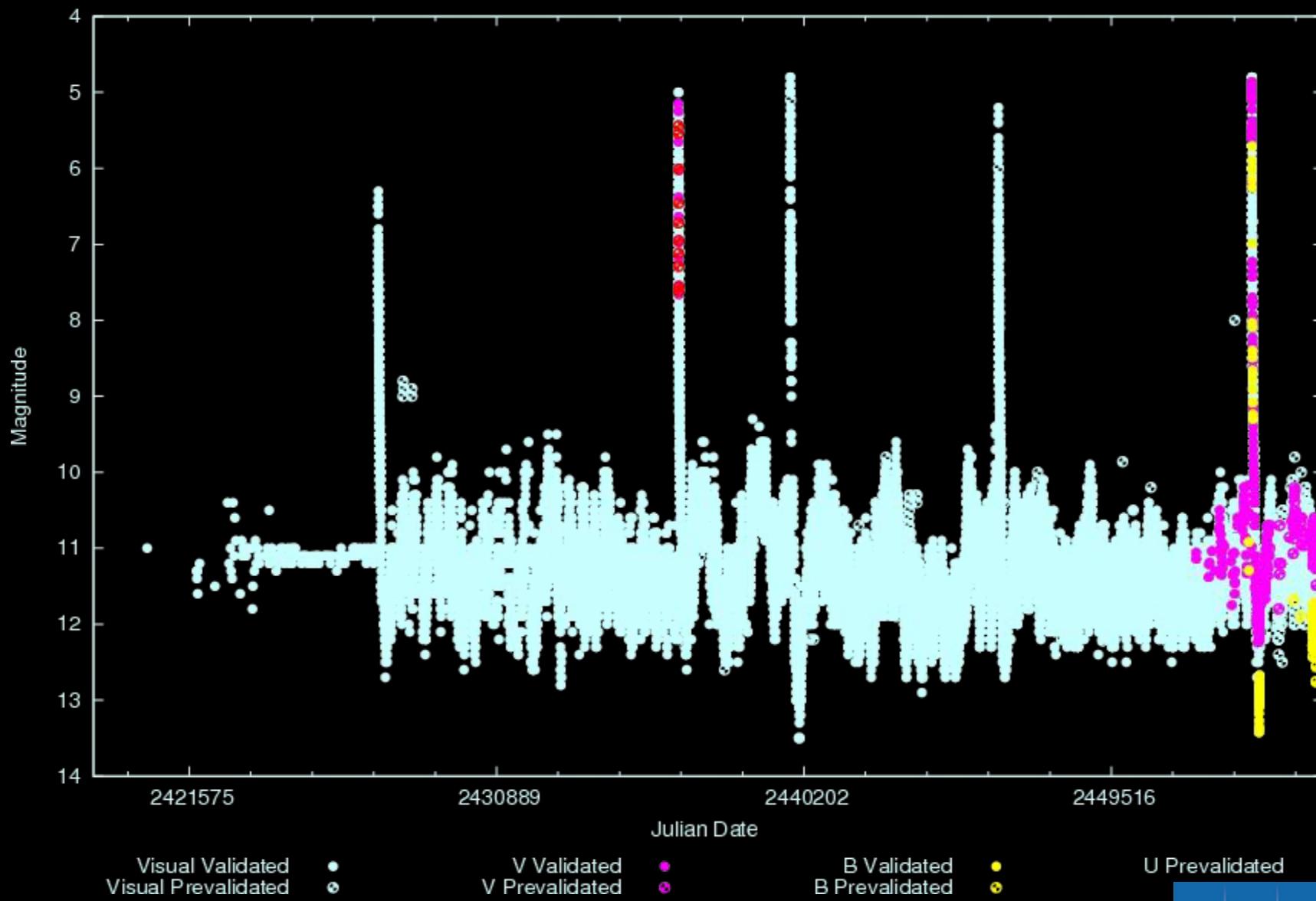
F. Paresce,  
R. Jedrzejewski  
(STScI),  
NASA/ESA

# GK Per



# RS Ophiuchi

AAVSO DATA FOR RS OPH - WWW.AAVSO.ORG



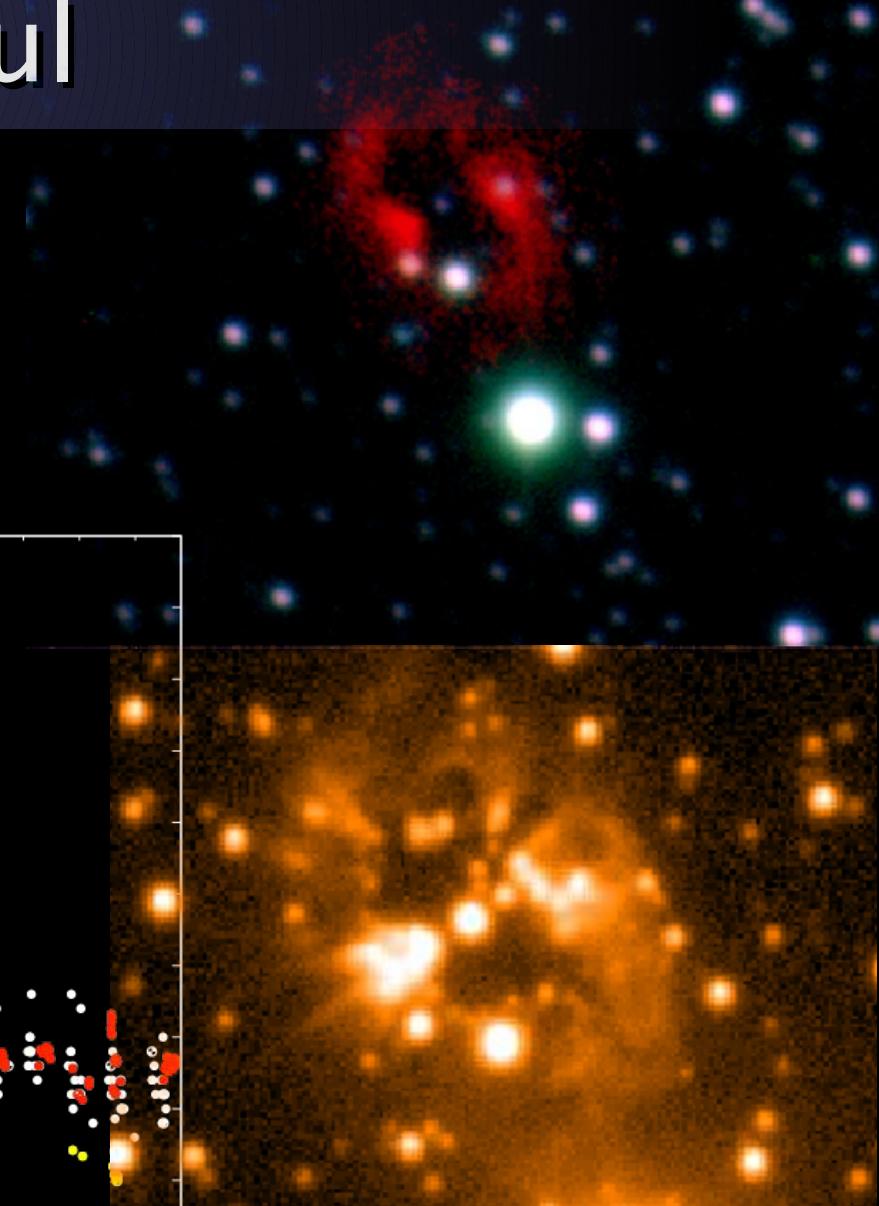
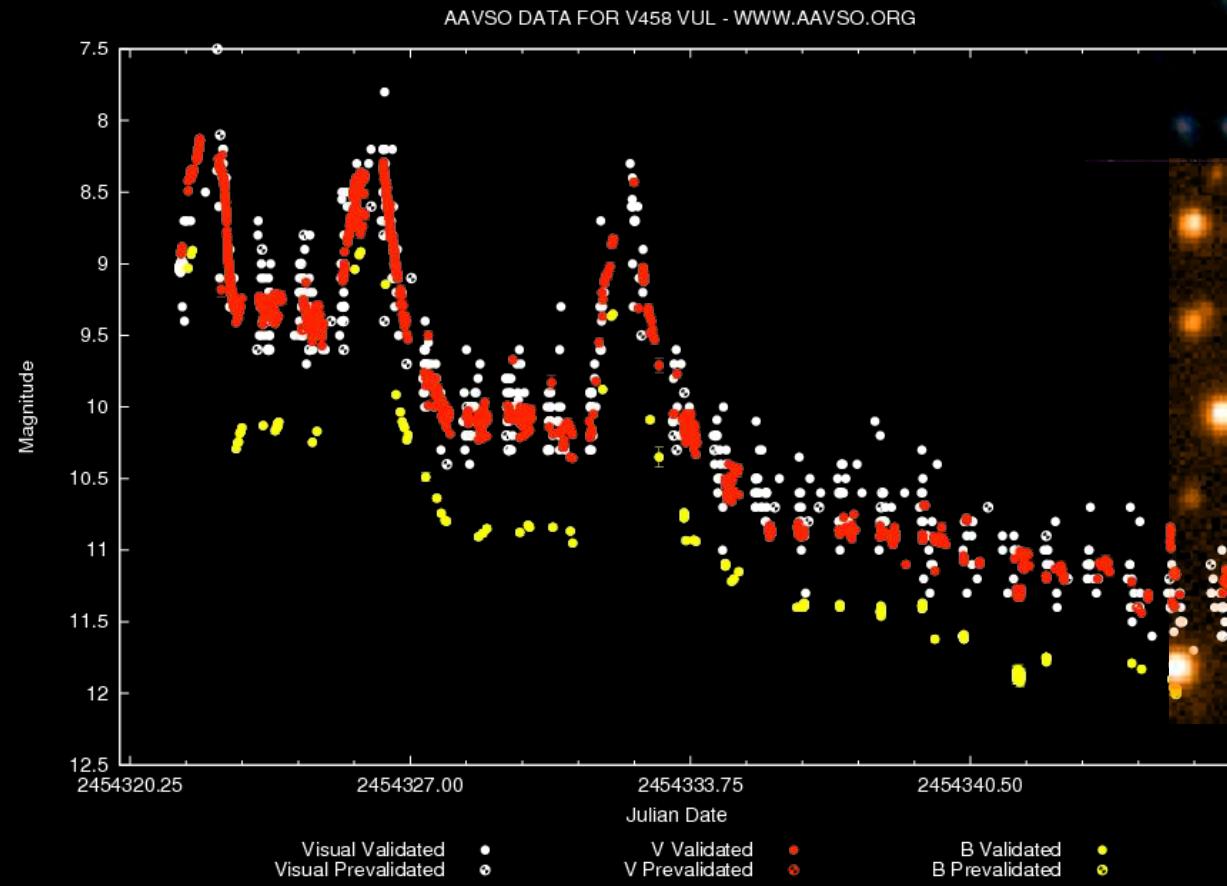
20 year period (1898, 1933, 1958, 1967, 1985, 2006)

Binary stars – Robert Izzard

Lecture 10

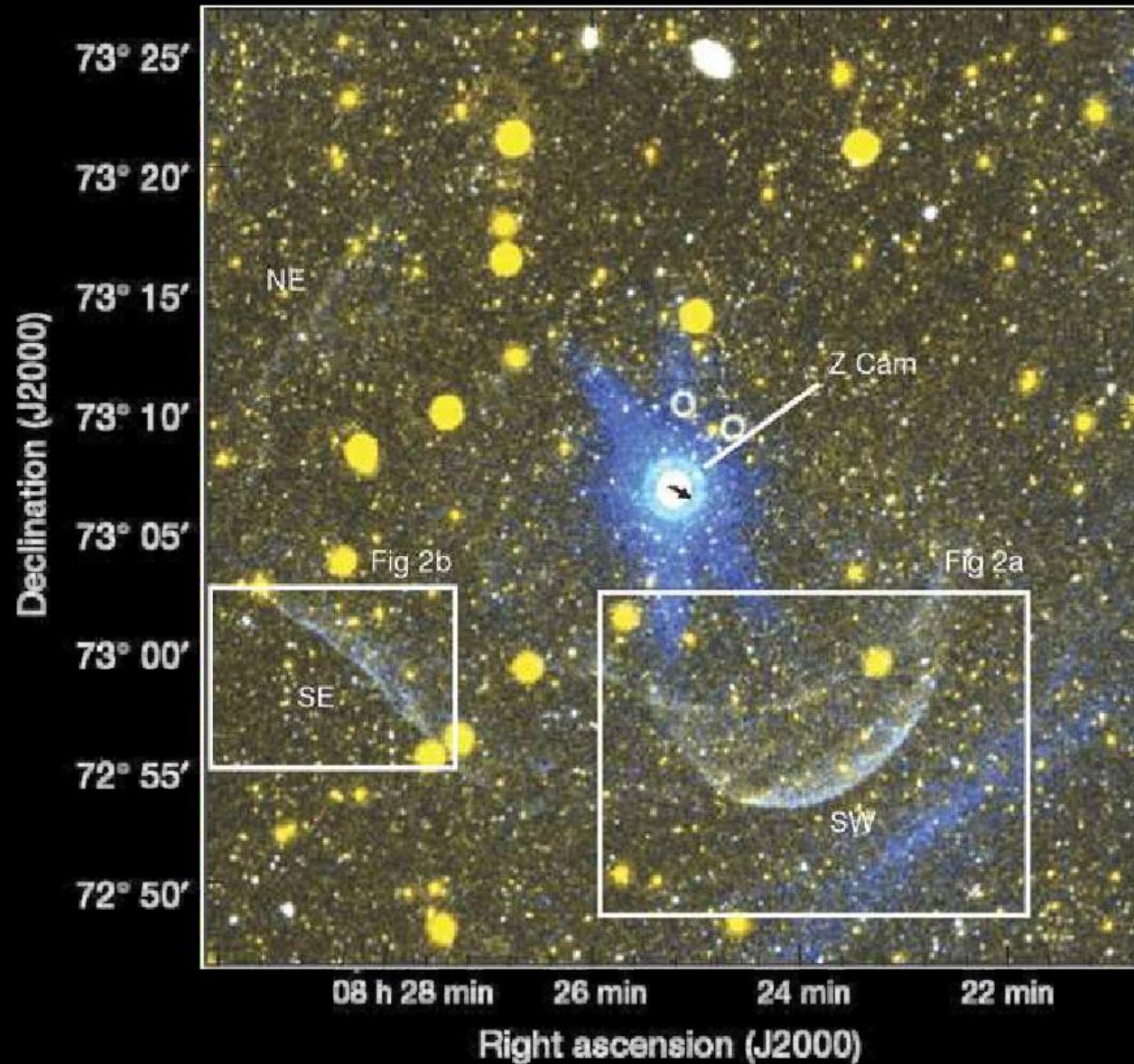
# V458 Vul

Nova inside a planetary nebula!  
PN Estimated 14000 years old  
Post common envelope  
SNIa progenitor???



Wesson et al 2008

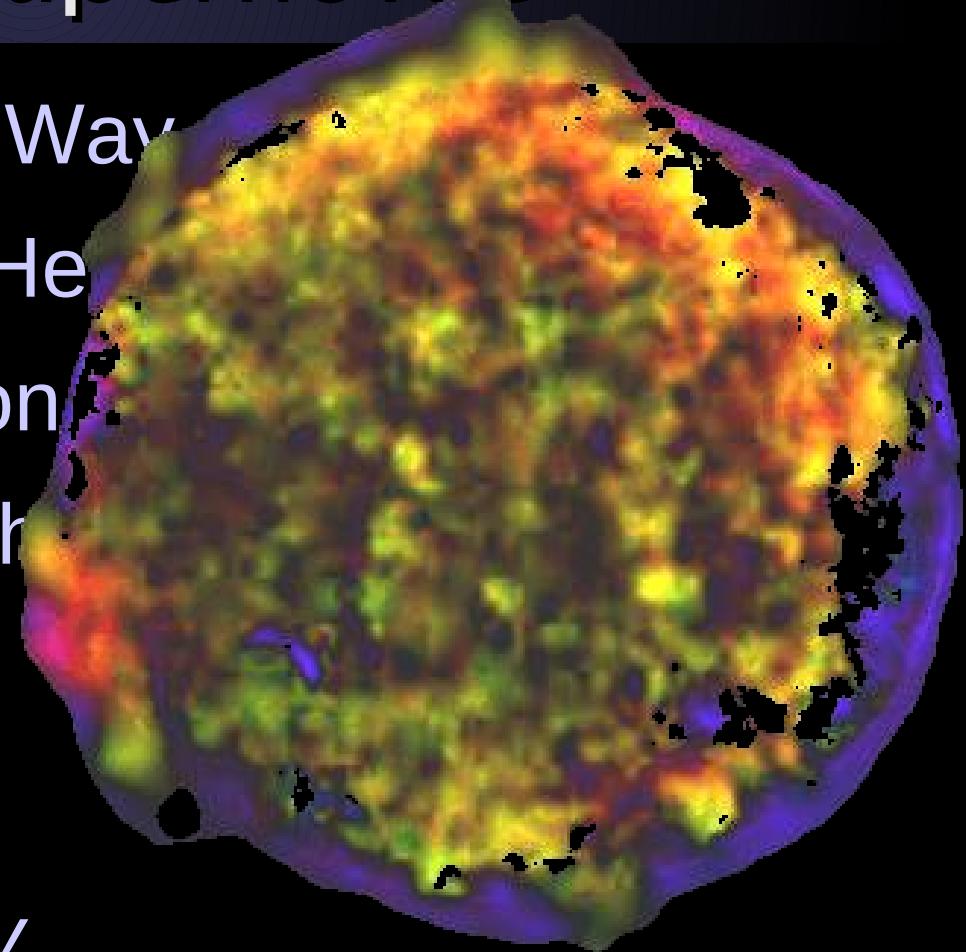
# Classical - Dwarf Connection



Shara et al 2007  
Nature 446, 159

# Type Ia supernovae

- 1/250 years in Milky Way
- Spectrum: Si; No H,He
- White dwarf explosion
- Sub-MCh/MCh/>MCh
- Mag  $M_V = -19$
- “Standard Candles”
- Useful for *cosmology*
- Iron-peak nucleosynthesis



Tycho's SN remnant  
NASA/MPIA/Calar Alto Observatory,  
Oliver Krause et al.

Fe, Ni, Co, Ti ...

# SN<sub>Ia</sub> lightcurve

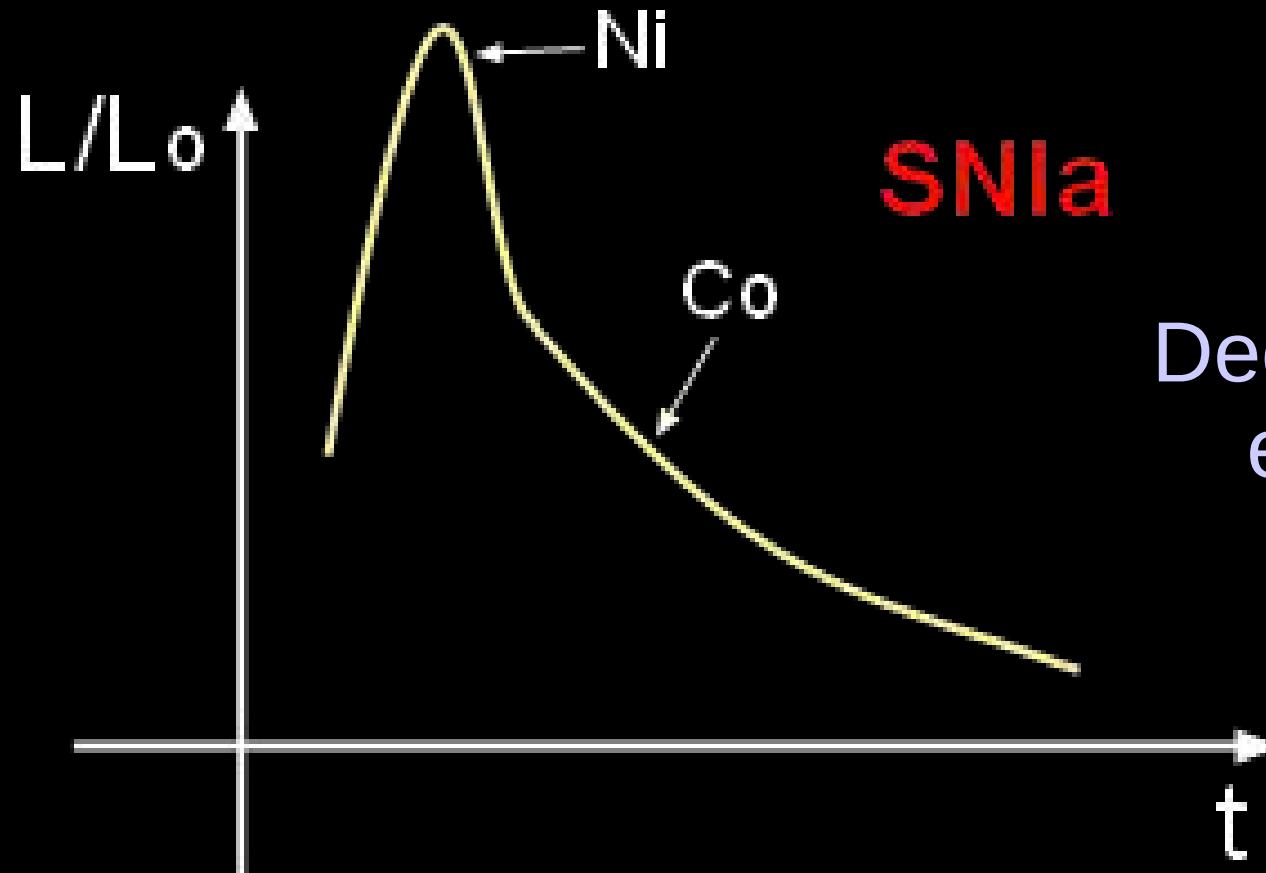
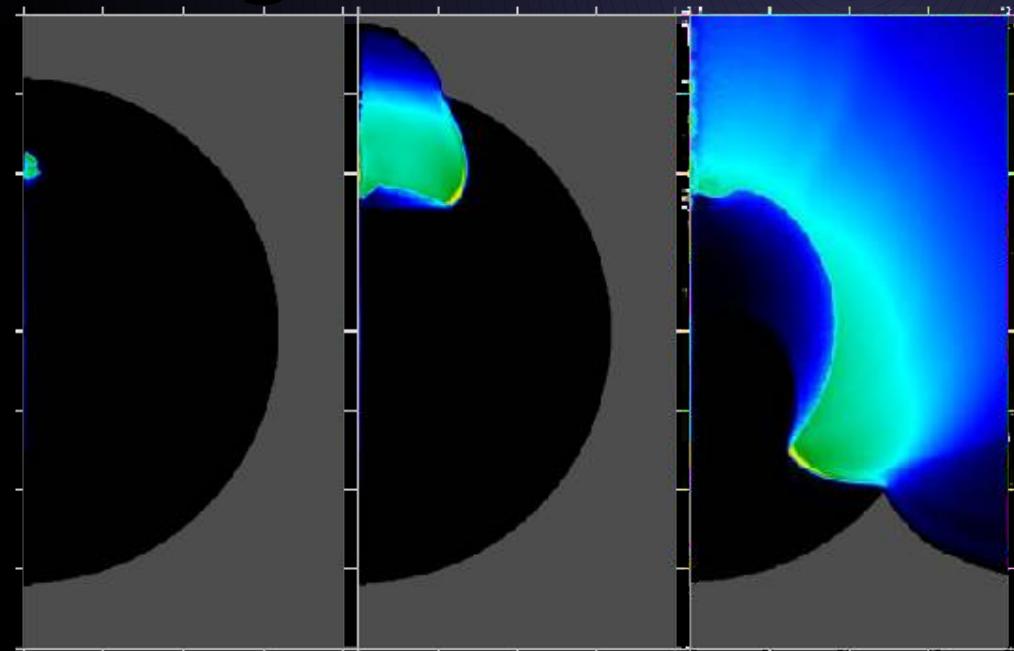


Image from Wikipedia

# Edge-Lit Detonation ( $M < 1.4$ )



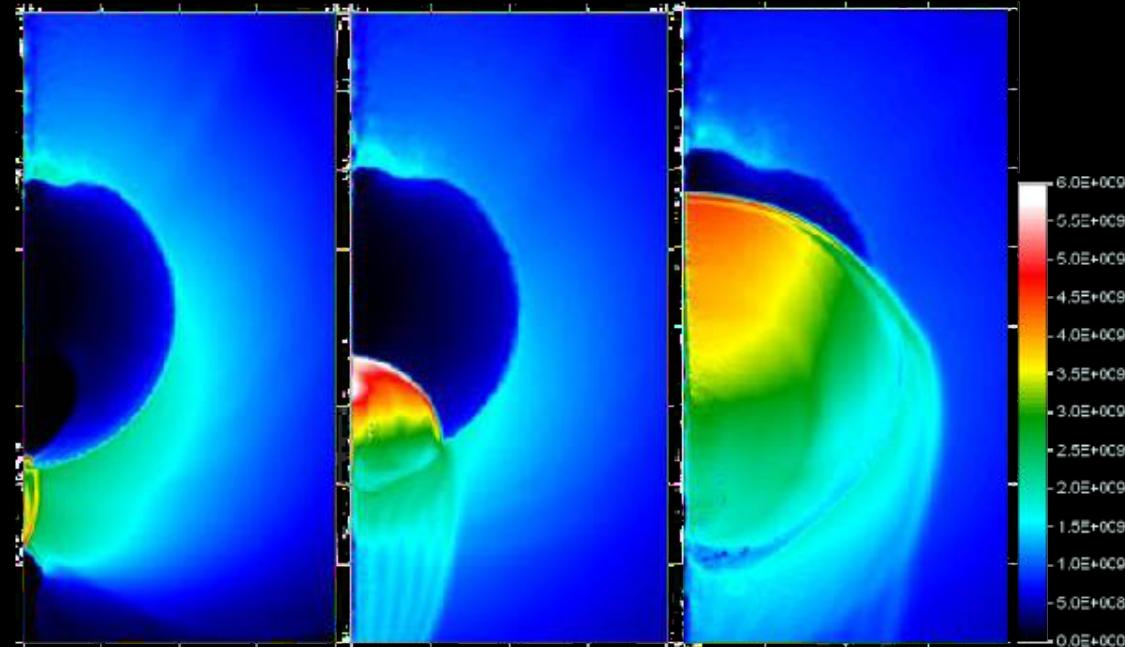
colour=temperature

white =  $6 \times 10^9$  K

$M_{\text{CO}} = 0.7 M_{\odot}$

$M_{\text{He}} = 0.2 M_{\odot}$

Total time < 2  
seconds!

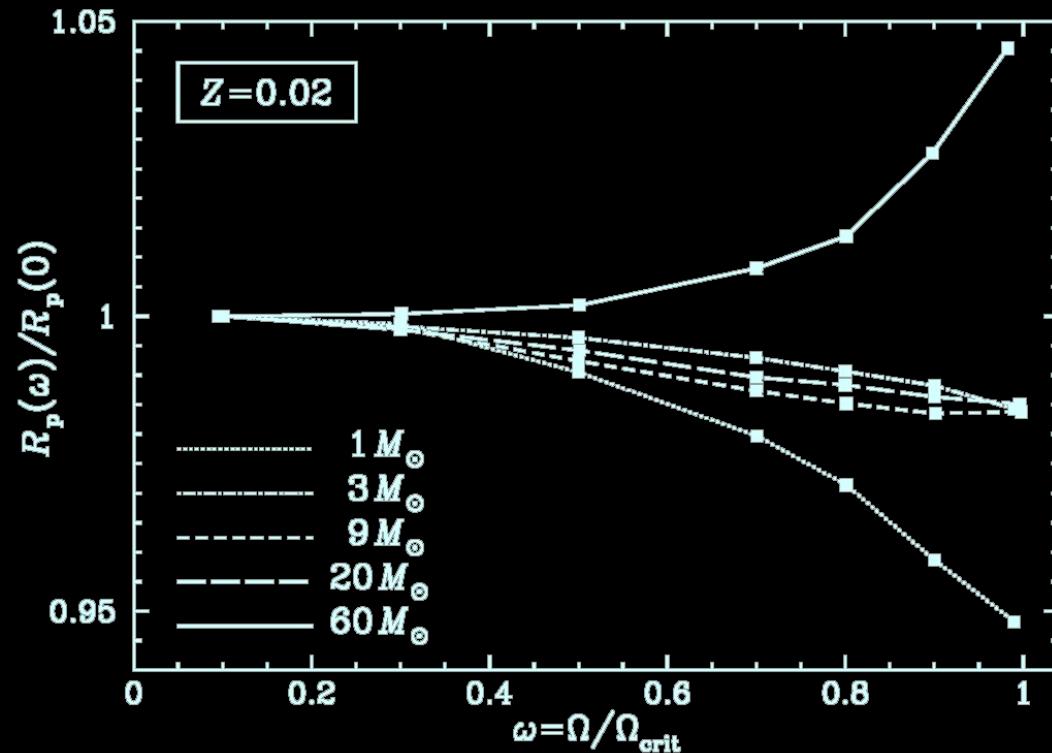


Forcada, Garcia-Senz,  
Jose 2007

# Pause for coffee

# An aside: dimensions of rotating stars

- Can we treat stars as essentially *single stars*?
- Polar radius is approx const.



Ekstrom et al  
2008 A&A  
478, 467

Variations in the polar radius as a function of the ratio  $\omega = \Omega/\Omega_{\text{crit}}$ , normalized to the non-rotating value, for various masses at standard metallicity.

# Rapid Stellar Models

- Creating *detailed* stellar models is slow and difficult
- Rapid or synthetic stellar models are faster
- Replace details solver with pre-solved model set:
  - Fitting formulae
  - Or lookup tables
- Sacrifice (usually unwanted) details for speed: up to 10,000,000 times faster.



# Fitting Formulae

- Eggleton, Fitchett, Tout 1989, Hurley et al 2000, 2002
- Zero-age main sequence:

$$L_0 = \begin{cases} \frac{1.107M^3 + 240.7M^9}{1 + 281.9M^4} & M \leq 1.093 \\ \frac{13990M^5}{M^4 + 2151M^2 + 3908M + 9536} & M \geq 1.093 \end{cases}$$

$$R_0 = \begin{cases} \frac{0.1148M^{1.25} + 0.8604M^{3.25}}{0.04651 + M^2} & M \leq 1.334 \\ \frac{1.968M^{2.887} - 0.7388M^{1.679}}{1.821M^{2.337} - 1} & M \geq 1.334 \end{cases}$$

# Fitting Formulae

- Time evolution function of  $\tau = t/t_{\text{MS}}$

$$t_{\text{MS}} = \frac{2550 + 669M^{2.5} + M^{4.5}}{0.0327M^{1.5} + 0.346M^{4.5}}.$$

- Then

$$\log_{10} L = \log_{10} L_0 + \alpha \tau_{\text{MS}} + \beta \tau_{\text{MS}}^2$$

$$\log_{10} R = \log_{10} R_0 + \alpha' \tau_{\text{MS}} + \beta' \tau_{\text{MS}} + \gamma' \tau_{\text{MS}}^3$$

# Fitting formulae

$$\alpha = \begin{cases} 0.2594 + 0.1348 \log_{10} M & M \leq 1.334 \\ 0.09209 + 0.05934 \log_{10} M & M > 1.334 \end{cases}$$

$$\beta = \begin{cases} 0.144 - 0.833 \log_{10} M & M \leq 1.334 \\ 0.3756 \log_{10} M - 0.1744 (\log_{10} M)^2 & M > 1.334 \end{cases}$$

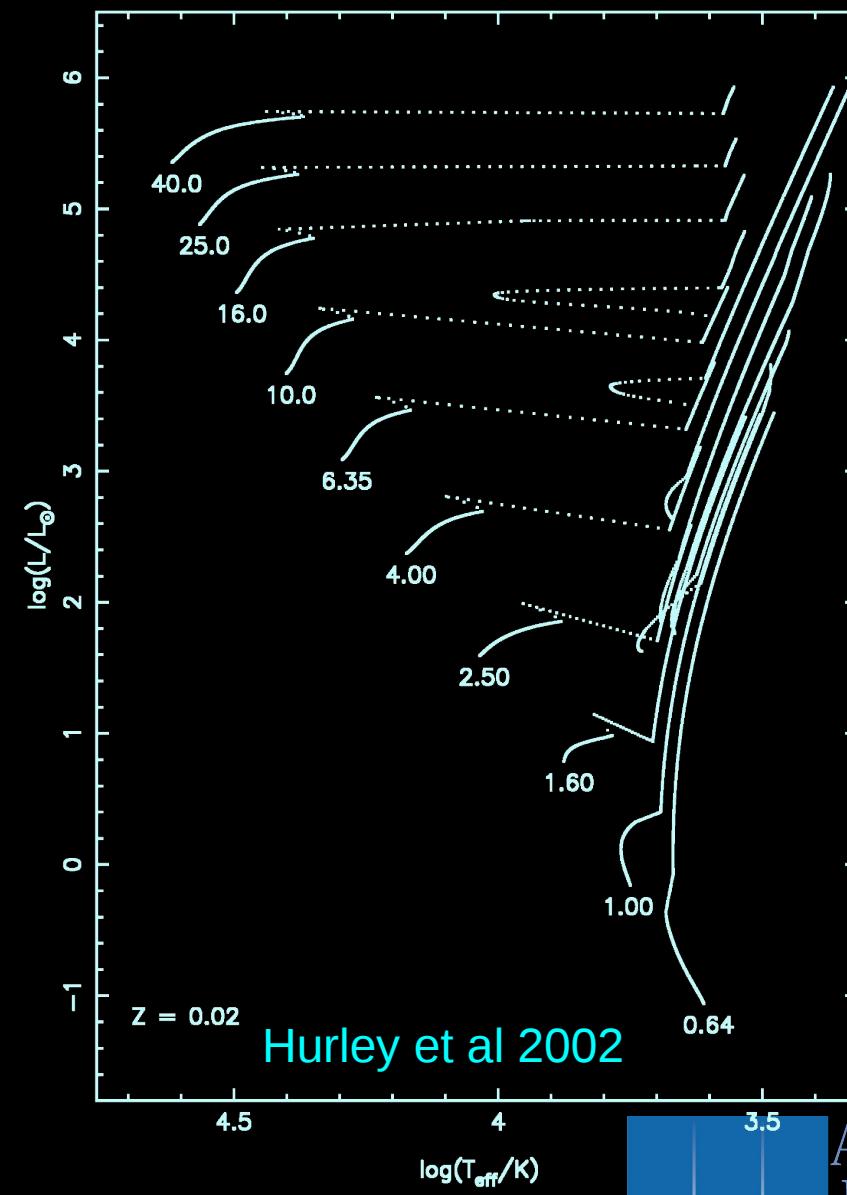
$$\alpha' = \begin{cases} 0 & M \leq 1.334 \\ 0.1509 + 0.1709 \log_{10} M & M > 1.334 \end{cases}$$

$$\beta' = \begin{cases} 0.2226 \log_{10} M & M \leq 1.334 \\ -0.4805 \log_{10} M & M > 1.334 \end{cases}$$

$$\gamma' = \begin{cases} 0.1151 & M \leq 1.334 \\ 0.5083 \log_{10} M & M > 1.334 \end{cases}.$$

Even more complicated formulae apply for later phases of evolution!  
But computers *do not care* ...

# Real vs Synthetic HRD



# Pros and Cons

## ■ Pros

- Faster to compute
- Stable

## ■ Cons

- Fixed input physics (but could use tables!)
- Discard of potentially useful information
- Off-grid treatment
- Fitting errors (<5%)

$$\begin{aligned}\log_{10} L &= \log_{10} L_0 + \alpha \tau_{\text{MS}} + \beta \tau_{\text{MS}}^2 \\ \log_{10} R &= \log_{10} R_0 + \alpha' \tau_{\text{MS}} + \beta' \tau_{\text{MS}} + \gamma' \tau_{\text{MS}}^3\end{aligned}$$

# Population Synthesis

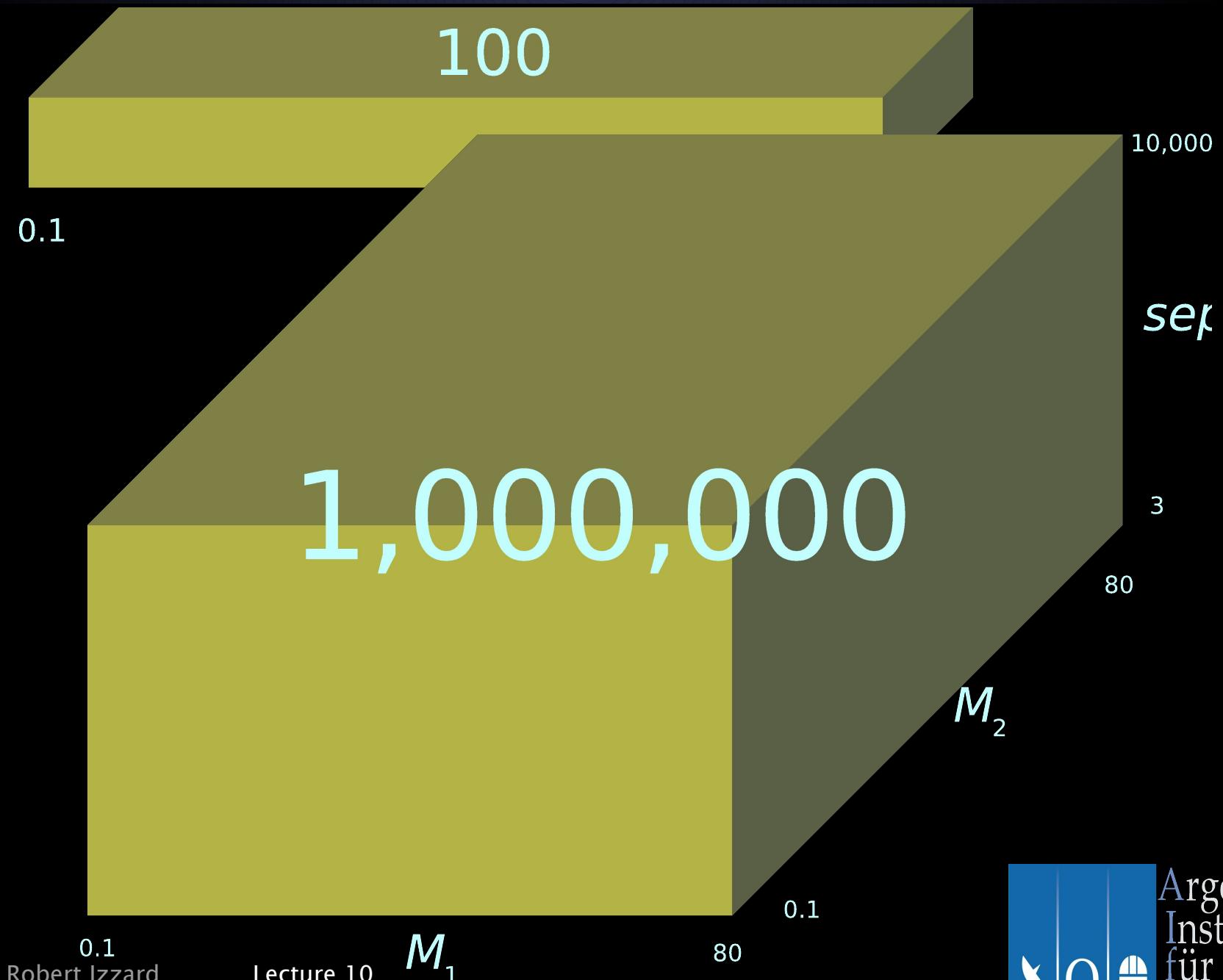
*The process of combining stellar models into a stellar population upon which meaningful statistical analysis can be performed and compared to observations to better constrain the underlying physics.*

1. Make your stellar models
2. Weight these according to mass, separation, time etc.
3. Extract simulated value(s)-compare
4. Determine the “real-life” distribution from obs.
5. Compare the two, see what's wrong
6. Refine your stellar models
7. Return to step 1 until you are happy  
  
(or funding runs out)

# The Parameter Space Problem

- To make a single star population, one parameter  
only: Mass  $\sim N \times \Delta t$
- Runtime is
- Binaries many parameters :
  - Primary mass  $M_1$
  - Secondary mass  $M_2$
  - Sep/Period  $a$  or  $P$
  - Eccentricity  $e$
  - Maybe more e.g.
- Runtime  $\sim N^3 \times \Delta t$

# Parameter Spaces



# Popsyn + rapid code



# Discretising Parameter Space

- Single Stars

$$\delta \ln M = \frac{\ln M_{\max} - \ln M_{\min}}{n}$$

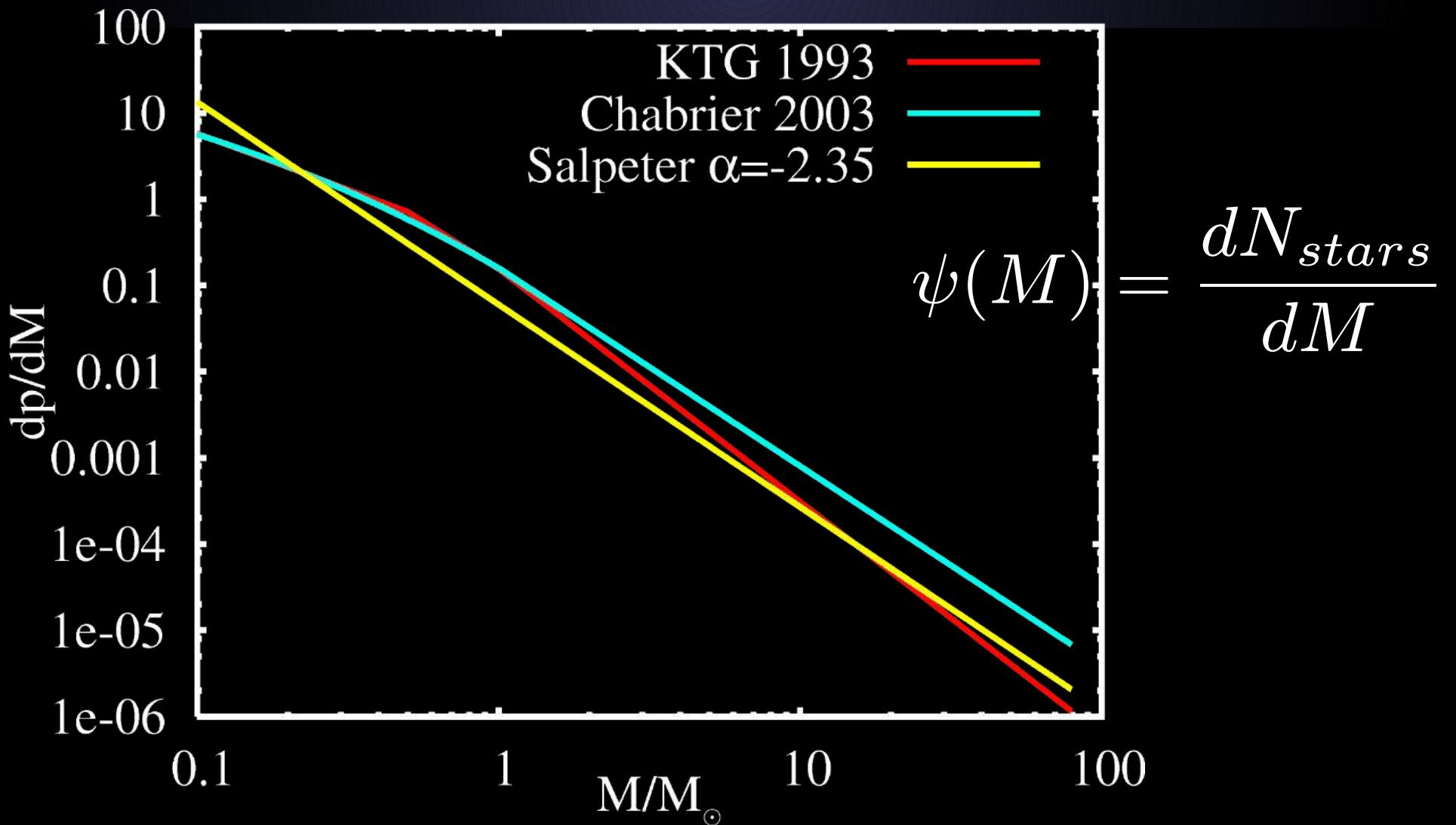
- Each star has a probability of existence

$$\delta p_i = \psi(M_i) \delta \ln M$$

- Where  $\psi$  is the *initial mass function*

$$\sum_i \delta p_i = 1$$

# IMF



Salpeter IMF  $\psi \propto M^{-2.35}$

# Discretising Parameter Space

- Binary Stars

$$\delta \ln x = \frac{\ln x_{\max} - \ln x_{\min}}{n_x}$$

where  $x$  is  $M_1, M_2, a, P, e \dots$

- Each star has a probability of existence

$$\delta p_i = \Psi_i(M_1, M_2, a) \delta V$$

- Where  $\Psi$  is the *initial distribution function*

# Initial Distribution Function

$$\Psi_i = \psi(M_{1i}) \phi(M_{2i}/M_{1i}) \chi(a_i)$$

$$\psi(M_1) = \psi(M)$$

$$\phi\left(q = \frac{M_1}{M_2}\right) = \text{constant}$$

$$\chi(a) \propto a^{-1}$$

$$\chi(\ln a) = \text{constant}.$$

$$\delta p_i = \Psi_i \delta V_i$$

$$\delta V = \delta \ln M_1 \delta \ln M_2 \delta \ln a$$

$$\sum_i \delta p_i = 1$$

# Stellar accounts

- Define

$$\begin{aligned}\delta(\text{phase}) &= 1 \quad \text{during the phase,} \\ &= 0 \quad \text{otherwise.}\end{aligned}$$

- Time a star spends in a phase of interest

$$\Delta t_i = \sum_{t=t_{\min}}^{t_{\max}} \delta(\text{phase at } t)_i \delta t$$

# Stellar accounts

- The number of stars in the phase is

$$\begin{aligned}\text{count} &= \sum_i S \delta p_i \Delta t_i \\ &= \sum_i S \delta p_i \sum_{t_{\min}}^{t_{\max}} \delta(\text{phase})_i \delta t\end{aligned}$$

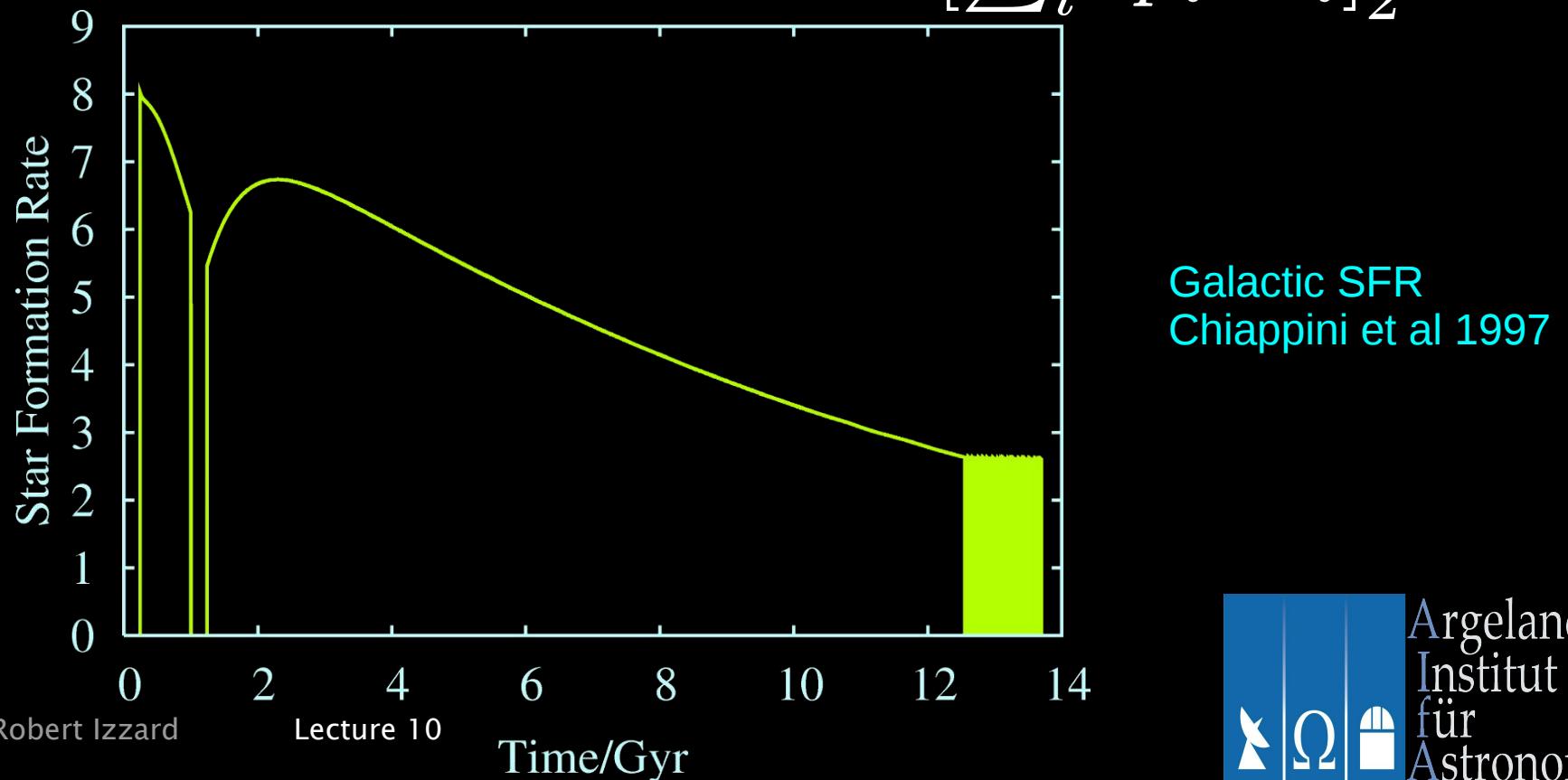
where  $S$  is the star formation rate

- In general we have to convolve a birth function with a star formation rate function
-

# Stellar accounts

- Simple case :  $S = \text{constant}$
- Divide counts to get ratios :  $S$  drops out

$$\text{ratio} = \frac{\left[ \sum_i \delta p_i \Delta t_i \right]_1}{\left[ \sum_i \delta p_i \Delta t_i \right]_2}.$$



# Stellar accounts

- The number of stars in the phase is

$$\sum_i S \delta p_i \Delta t_i$$

where  $S$  is the star formation rate

- In general we have to convolve a birth function with a star formation rate function

$$\sum_{t'_{\min}}^{t'_{\max}} \sum_i S(t) \delta p_i \delta(\text{phase at } t')_i \delta t'$$

# Compare to Observations

- Statistics!

- Boring (but not for everyone!)
- Necessary e.g.  $\chi^2$ , KS tests
- Key to good science



- Beware observational selection effects

- Often very hard to model
- Data combined from multiple surveys  
might be impossible to model!
- Sometimes whole papers are wrong  
because they neglect this!  
(not deliberately)

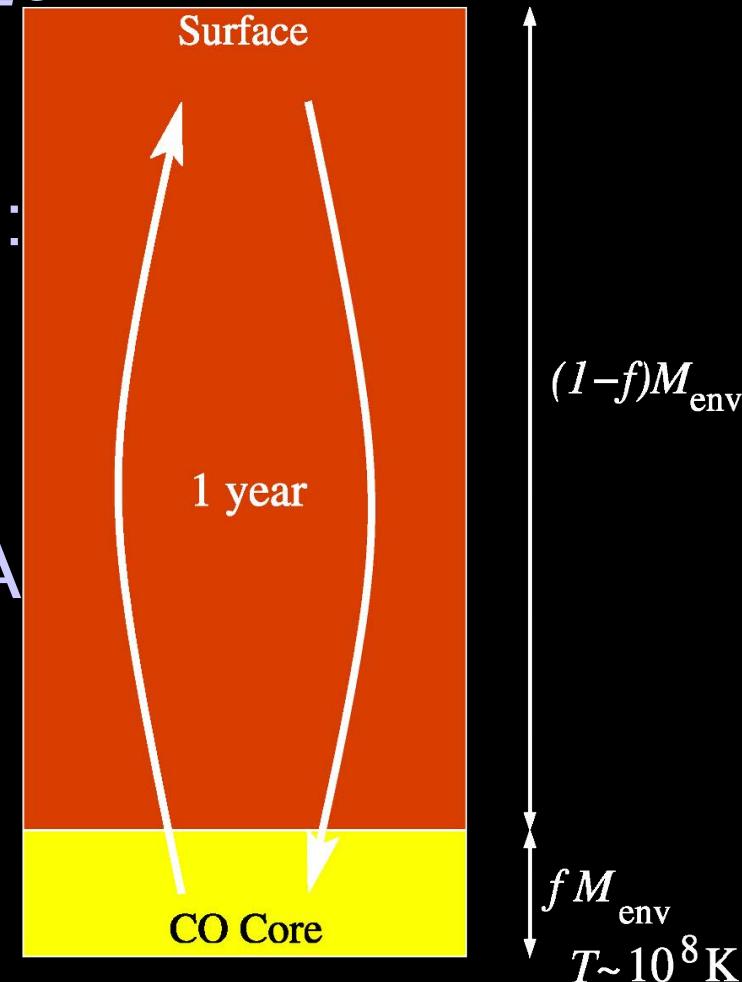


# A rapid code: *binary\_c*

- My code, my lectures, so ...
- Based on *SSE/BSE* of EFT89, Hurley et al 2000, 2002  
(e.g. see prev. eqs)
- Has fitting functions for stellar evolution
- +orbital algorithm: RLOF, Wind, Tides
- Common env., Novae, SNe Ia, Mergers etc.
- Online
- <http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cgi>

# *binary\_c/nucsyn*

- Added *nucleosynthesis* to *binary\_c*
- First and second dredge up
- TPAGB based on Karakas' models:
  - Third dredge up
  - Hot-bottom burning  
(CNO, NeNa, MgA)
  - S-process (Torino group)
- SN II/Ibc yields, novae
- Thermohaline mixing
- Physics updates over last few years



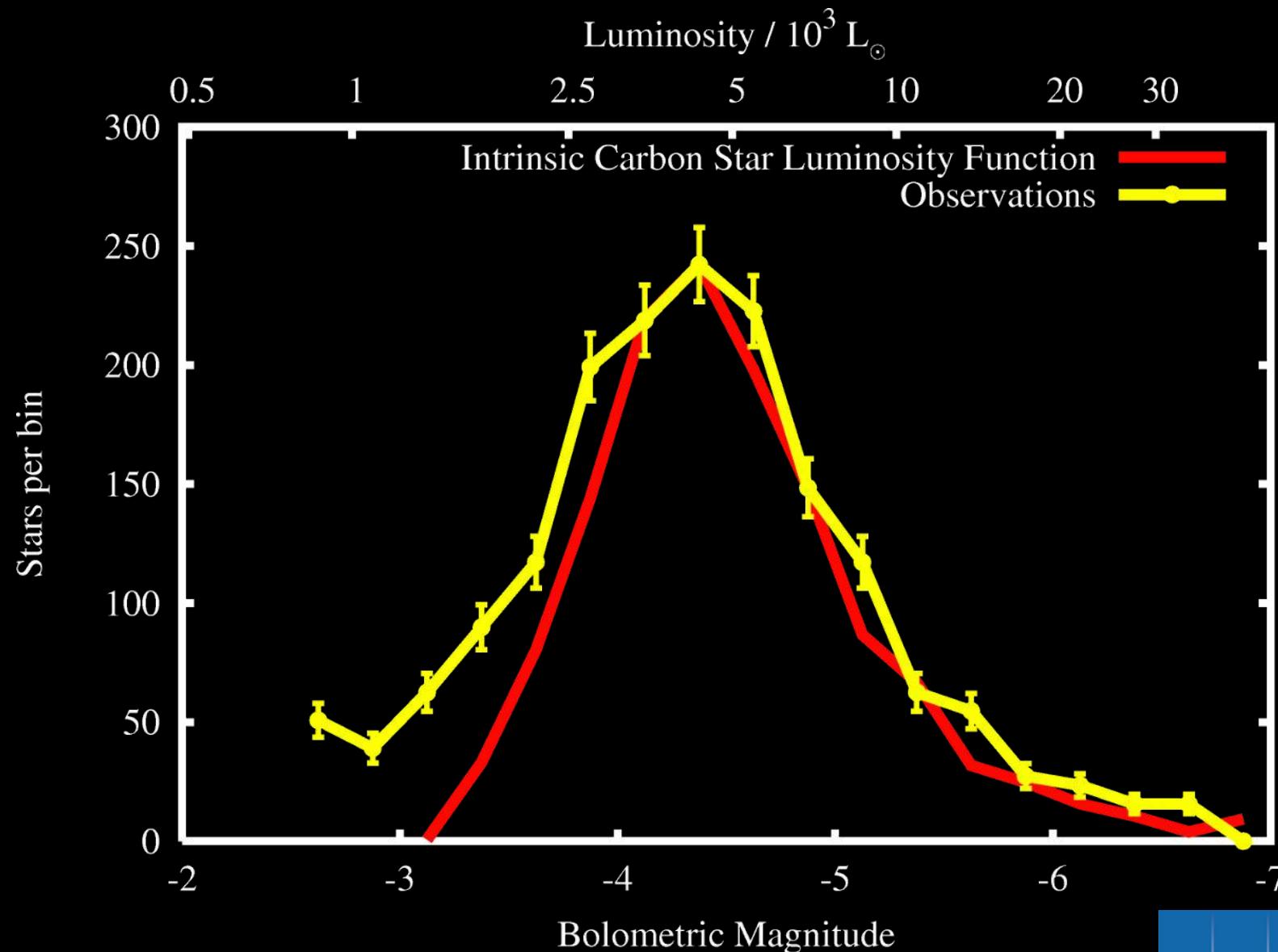
# Some examples of binary\_c

- Remember to try it yourself!
- <http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cgi>

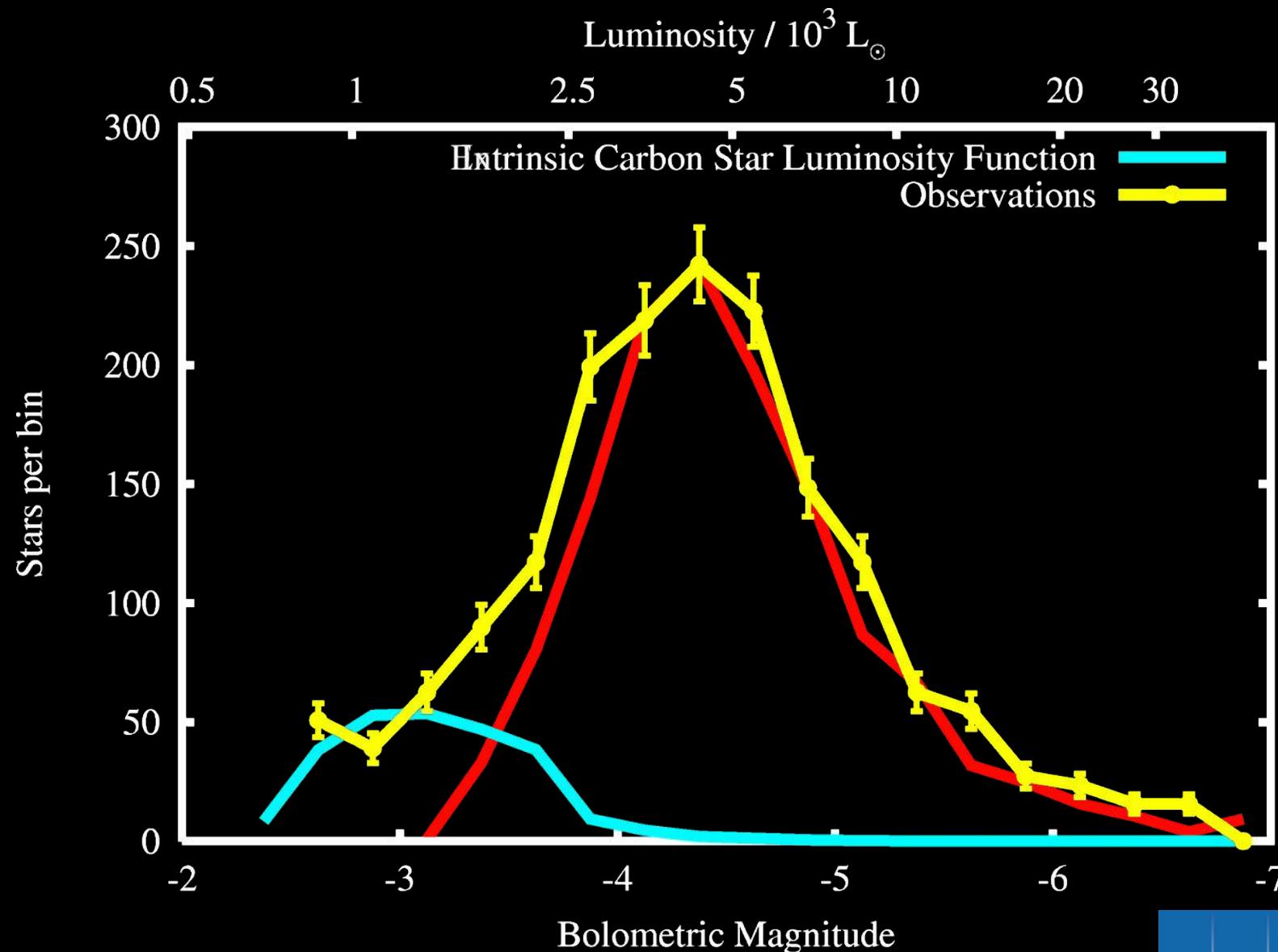
binary_c/nucsyn results										
A frontend to the <a href="#">binary_c/nucsyn code</a>										
Evolution Time (Myr)	Star 1 mass ( $M_{\odot}$ )	Star 2 mass ( $M_{\odot}$ )	Star 1 type	Star 2 type	Separation ( $R_{\odot}$ )	Period	Eccentricity	Star 1 R/ROL	Star 2 R/ROL	What's happening?
0.0000	14.000	6.000	Main Sequence	Main Sequence	100.000	25.92	0.00	0.106	0.095	In the beginning there was a star...
14.0936	13.718	6.002	Hertzsprung Gap	Main Sequence	101.340	26.63	0.00	0.256	0.103	Stellar Type Change
14.1165	13.715	6.003	Hertzsprung Gap	Main Sequence	101.384	26.64	0.00	1.000	0.103	Begin Roche Lobe Overflow
14.1165	13.715	6.003	Hertzsprung Gap	Main Sequence	101.384	26.64	0.00	1.000	0.103	Common Envelope Evolution in
14.1165	3.349	6.003	Main Sequence Naked Helium star	Main Sequence	12.748	1.72	0.00	1.000	0.103	Common Envelope Evolution
14.1165	3.349	6.003	Main Sequence Naked Helium star	Main Sequence	12.748	1.72	0.00	0.112	0.591	End of Roche Lobe Overflow
16.1738	3.042	6.014	Hertzsprung Gap Naked Helium star	Main Sequence	13.359	1.88	0.00	0.103	0.562	Stellar Type Change
16.3312	2.978	6.023	Hertzsprung Gap Naked Helium star	Main Sequence	13.397	1.89	0.00	1.003	0.559	Begin Roche Lobe Overflow

Binary stars

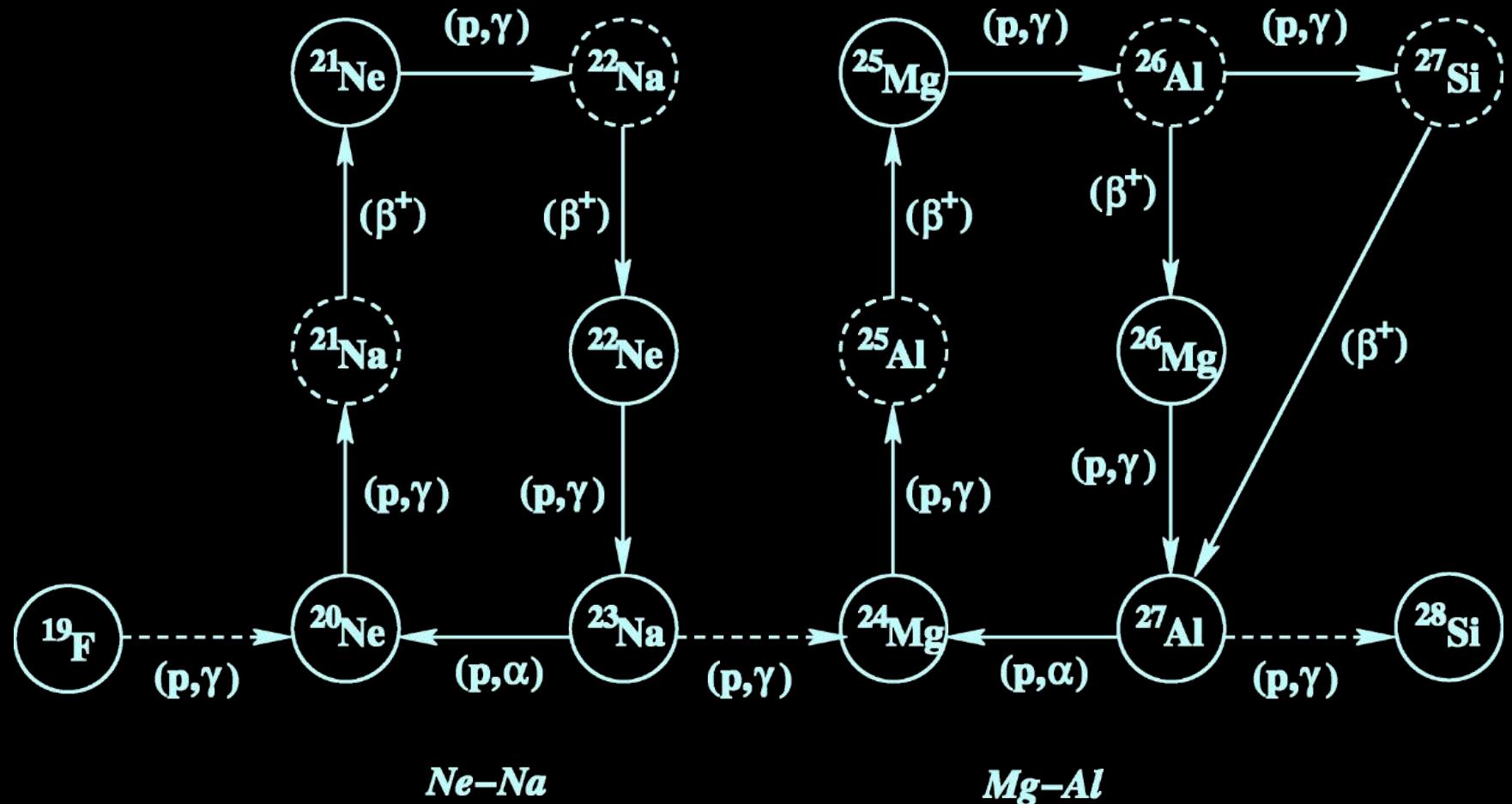
# Low-L Carbon Stars



# Low-L Carbon Stars



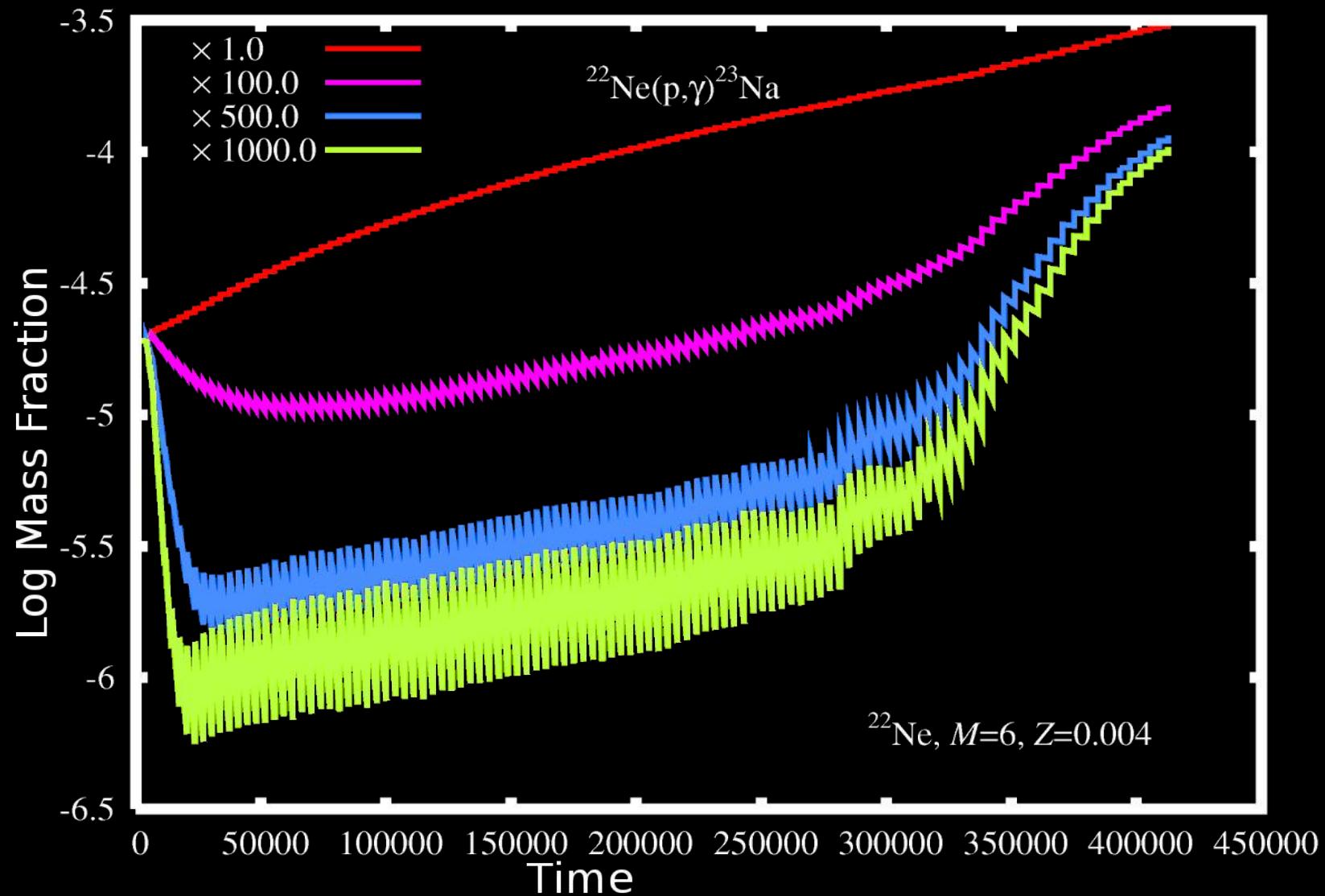
# Nuclear Burning Rates



# Nuclear Burning Rates

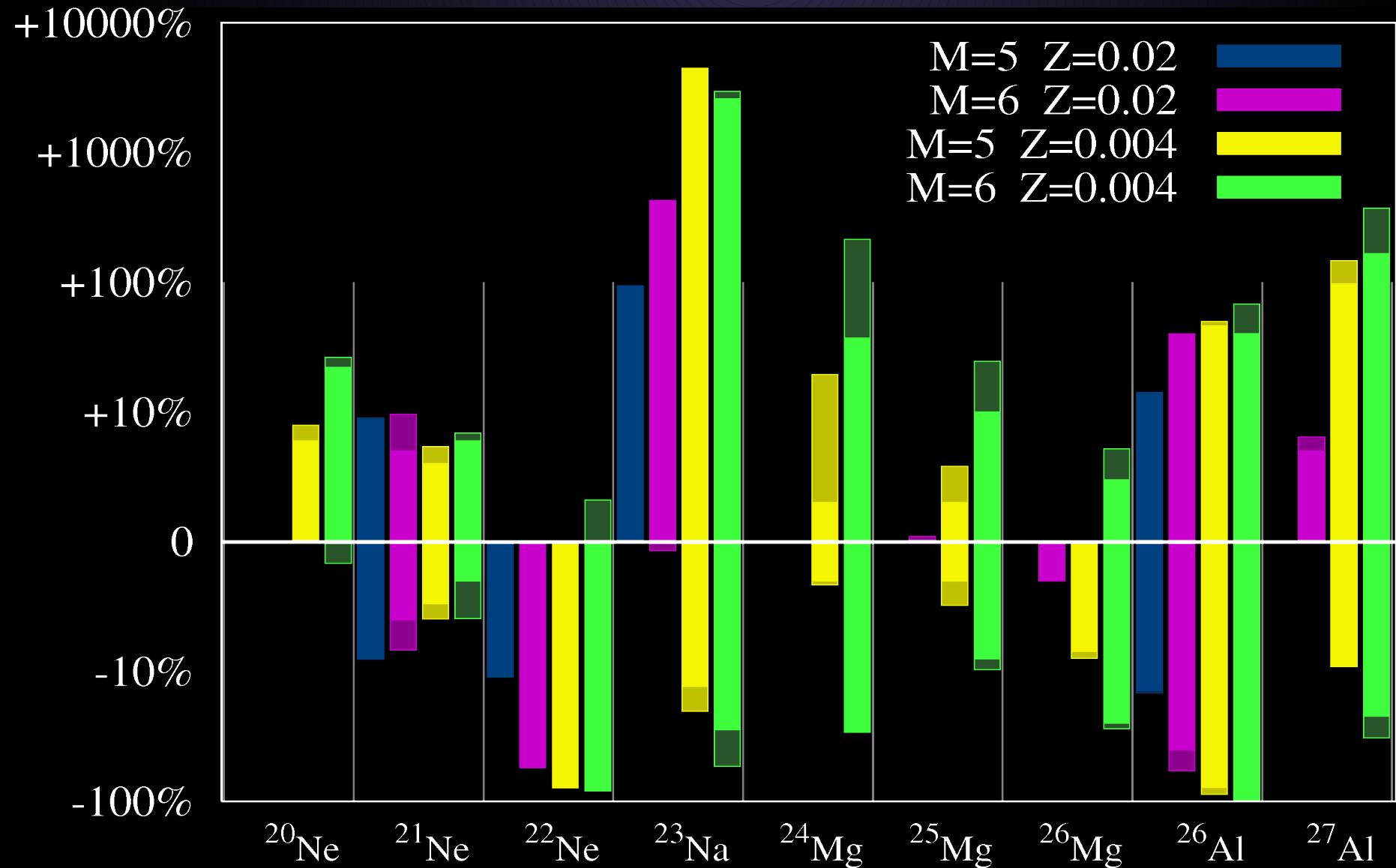
Rate			Source
$^{20}\text{Ne}(p, \gamma)^{21}\text{Na}(\beta^+)^{21}\text{Ne}$	-50%	+50%	NACRE
$^{21}\text{Ne}(p, \gamma)^{22}\text{Na}(\beta^+)^{22}\text{Ne}$	-20%	+20%	Iliadis et al. 2001
$^{22}\text{Ne}(p, \gamma)^{23}\text{Na}$	-50%	$\times 2000$	Hale et al. 2001
$^{23}\text{Na}(p, \alpha)^{20}\text{Ne}$	-30%	+30%	Rowland et al. 2004
$^{23}\text{Na}(p, \gamma)^{24}\text{Mg}$	/40	$\times 10$	Rowland et al. 2004
$^{24}\text{Mg}(p, \gamma)^{25}\text{Al}(\beta^+)^{25}\text{Mg}$	-17%	+20%	Powell et al. 1999
$^{25}\text{Mg}(p, \gamma)^{26}\text{Al}(\beta^+)^{26}\text{Mg}$	-50%	$\times 1.5$	Iliadis et al. 2001
$^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$	/4	$\times 10$	Iliadis et al. 2001
$^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$	-25%	$\times 3$	Iliadis et al. 2001
$^{26}\text{Al}(p, \gamma)^{27}\text{Si}$	/2	$\times 600$	Iliadis et al. 2001

# Nuclear Burning Rates



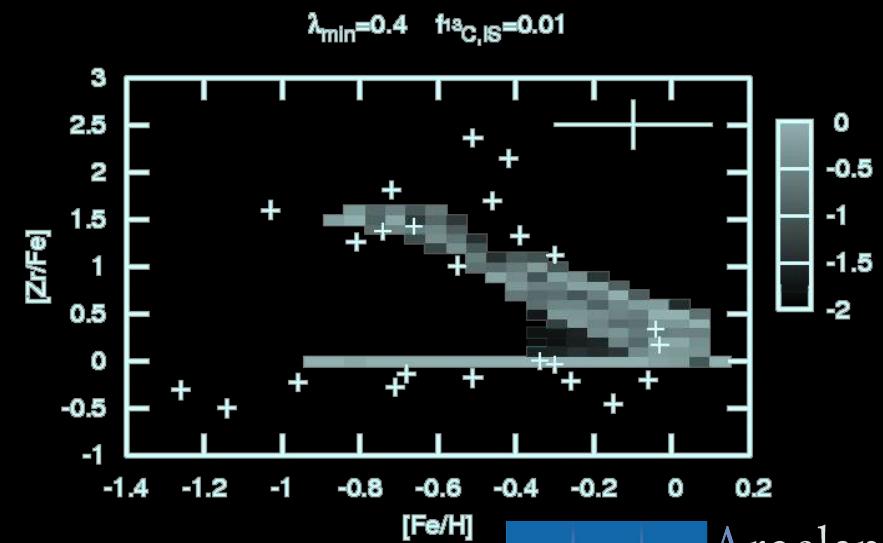
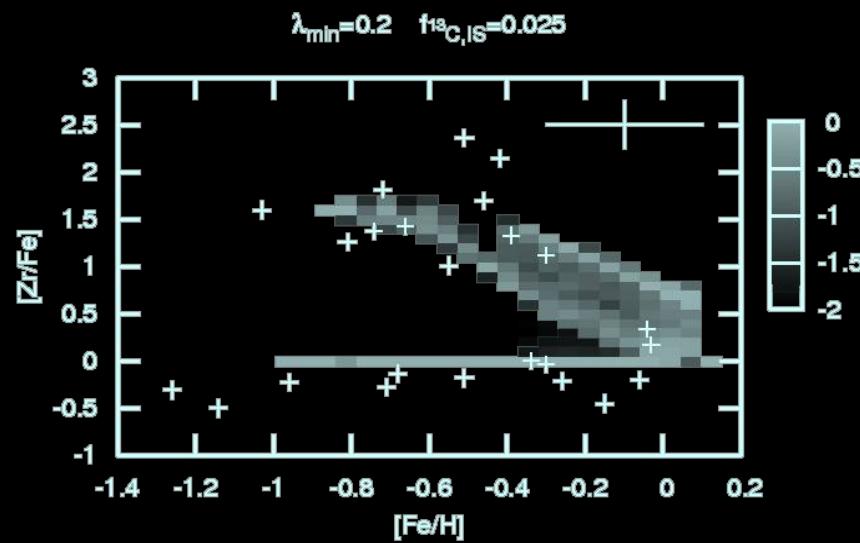
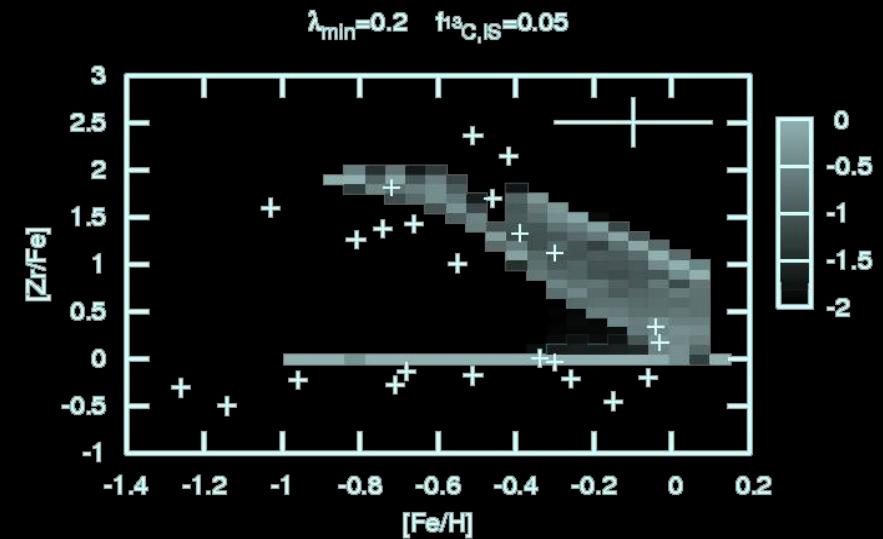
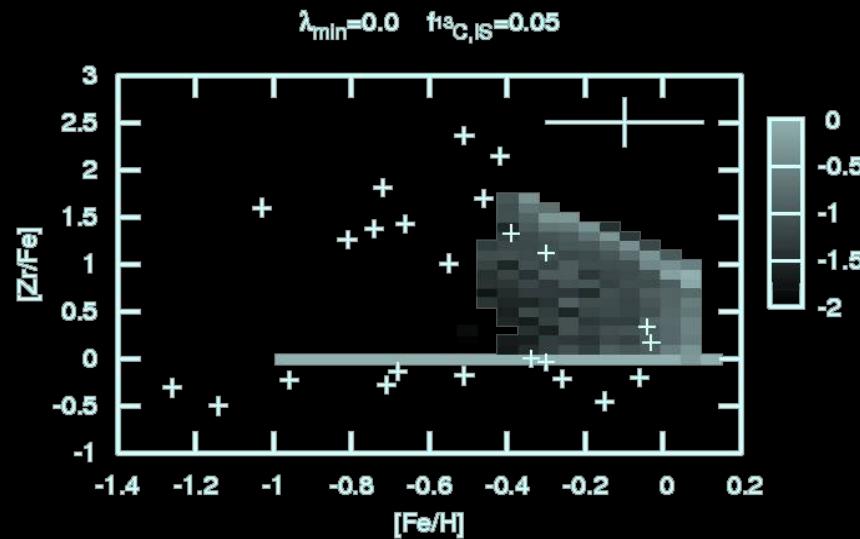
Izzard et al. 2007

# Nuclear Burning Rates



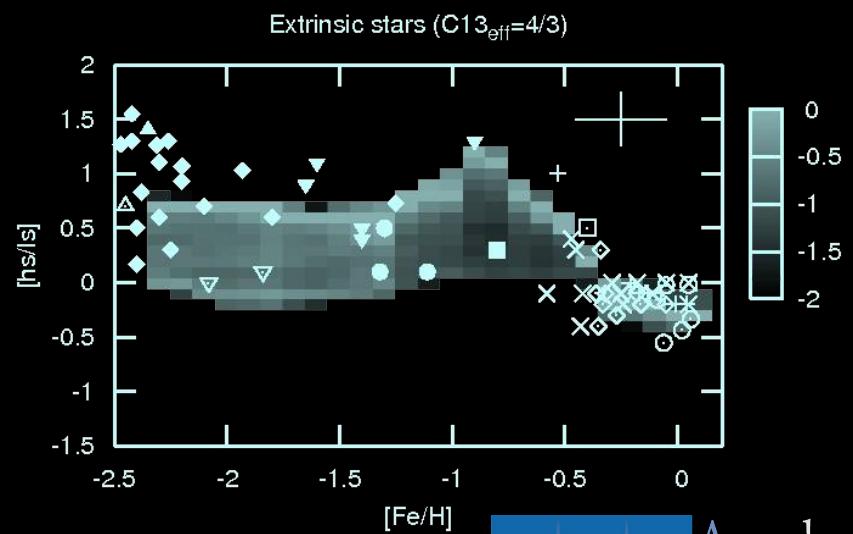
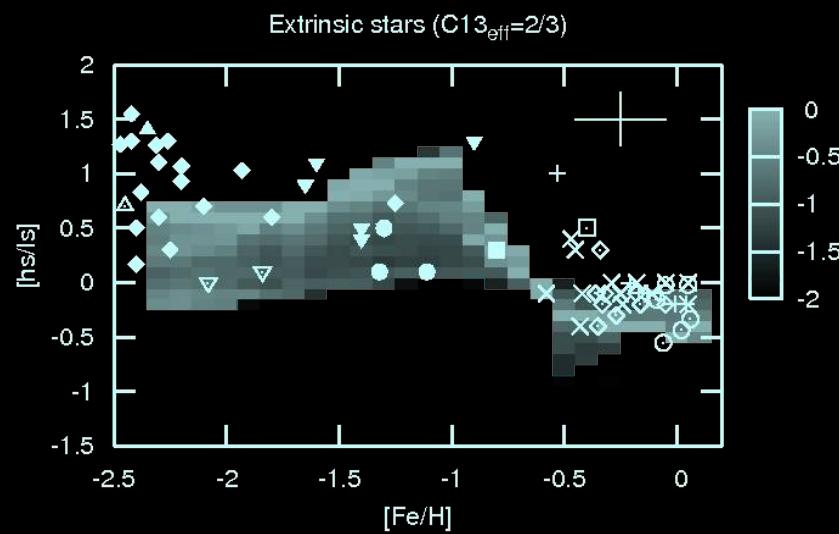
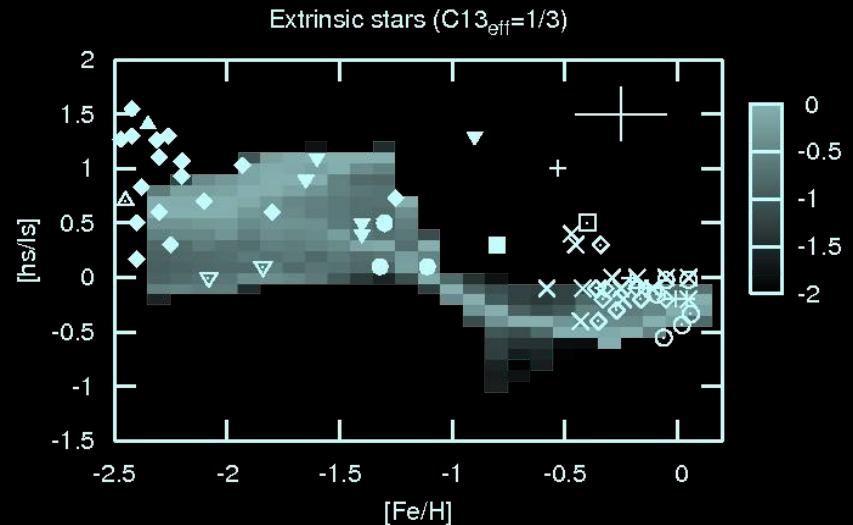
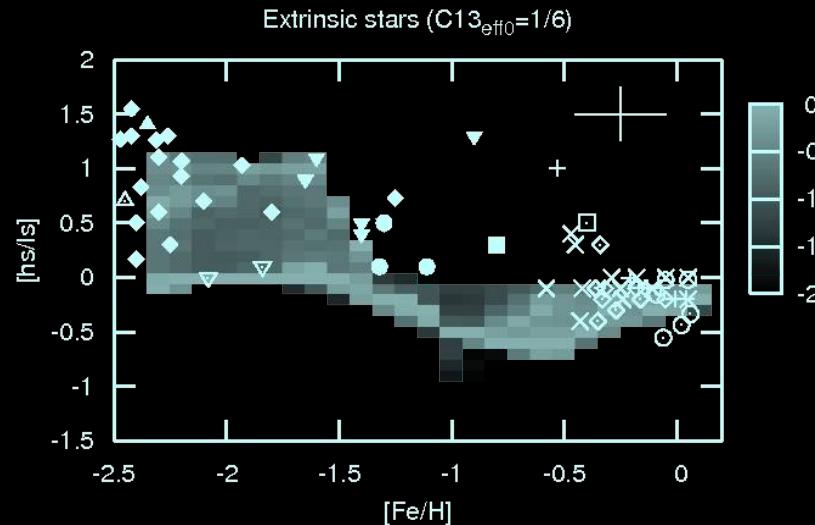
Izzard et al. 2007

# S-process in post-AGB



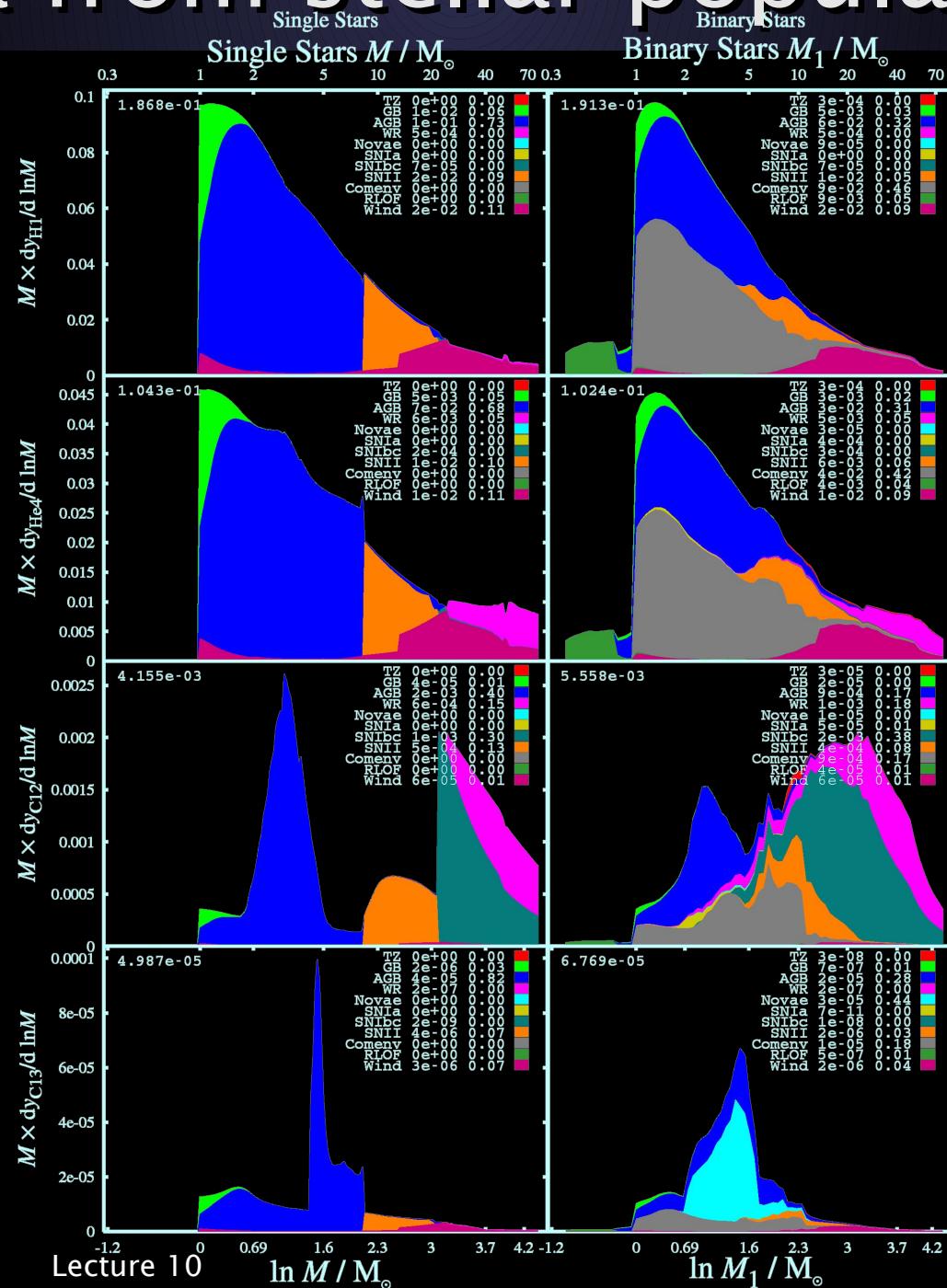
Bonacic et al. 2007

# S-process in post-AGB



Bonacic et al. 2007

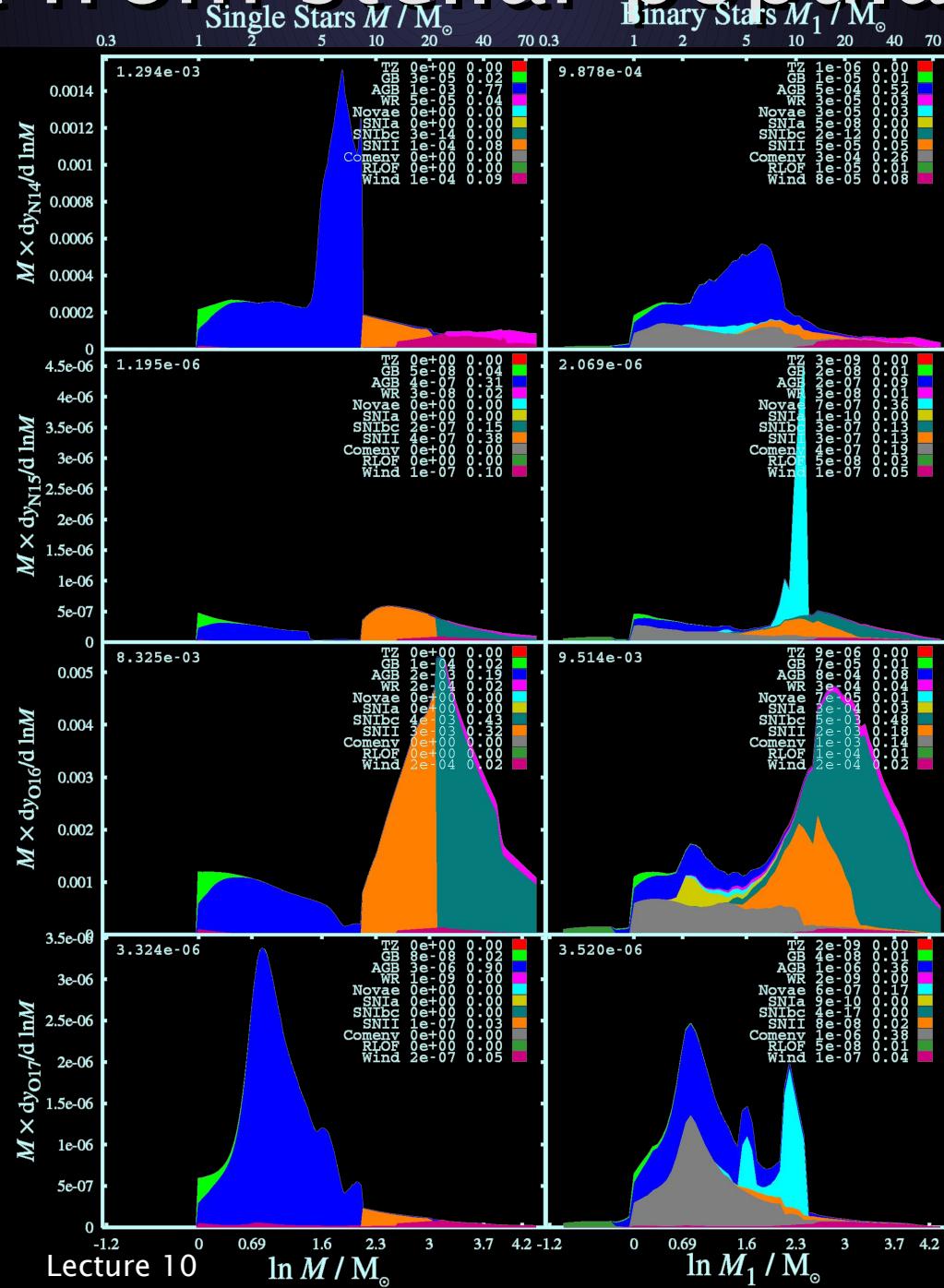
# Ejecta from stellar populations



Izzard PhD!

Binary stars – Robert Izzard

# Ejecta from stellar populations



Izzard PhD!

Binary stars – Robert Izzard

Lecture 10

$\ln M / M_{\odot}$

$\ln M_1 / M_{\odot}$