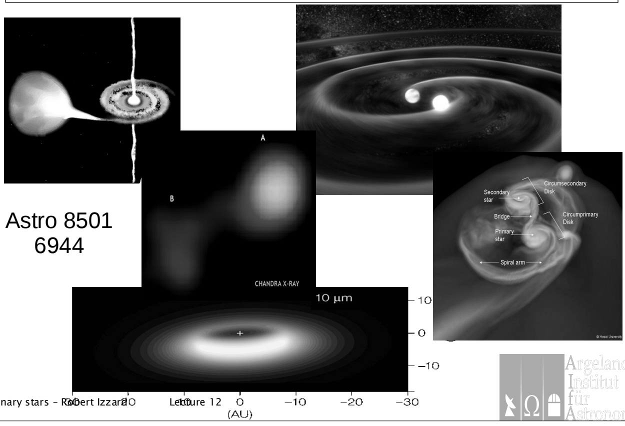
Binary Stars – Final Lecture!



Astro 8501
6944

CN cycle

$$\frac{d}{dt}\begin{bmatrix} ^{12}\mathrm{C} \\ ^{13}\mathrm{C} \\ ^{14}\mathrm{N} \end{bmatrix} = \begin{bmatrix} -1/\tau_{12} & 0 & 1/\tau_{14} \\ 1/\tau_{12} & -1/\tau_{13} & 0 \\ 0 & 1/\tau_{13} & -1/\tau_{14} \end{bmatrix} \begin{bmatrix} ^{12}\mathrm{C} \\ ^{13}\mathrm{C} \\ ^{14}\mathrm{N} \end{bmatrix}$$

$$\frac{d}{dt}\mathsf{U} = \Lambda \mathsf{U}$$

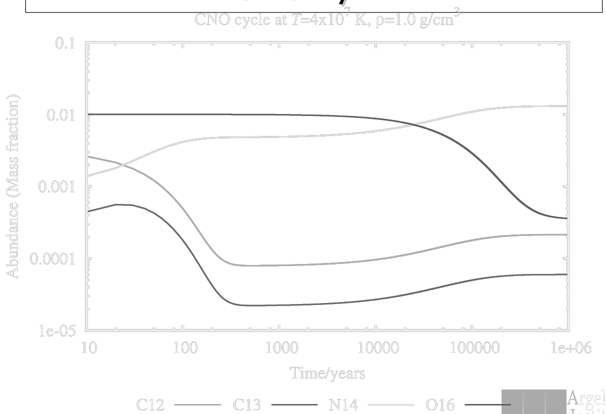
$$\mathsf{U}(t) = Ae^{\lambda_1 t}\mathsf{U}_1 + Be^{\lambda_2 t}\mathsf{U}_2 + Ce^{\lambda_3 t}\mathsf{U}_3$$

And similarly for the other cycles
See e.g. Clayton's book

Binary-star Nucleosynthesis

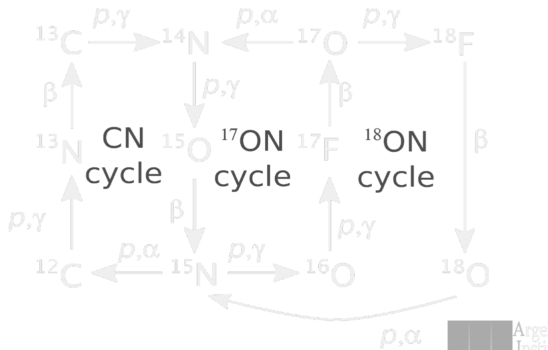
- Many types of stars are only or mostly found in binaries
- The physics we have learned about in this course will help us to understand them
- Chemically peculiar binaries:
  - Algols
  - Massive stars (WR stars etc)
  - Ba/CH/CEMP stars
  - Thermohaline mixing
  - Galactic chemical evolution

CNO cycle



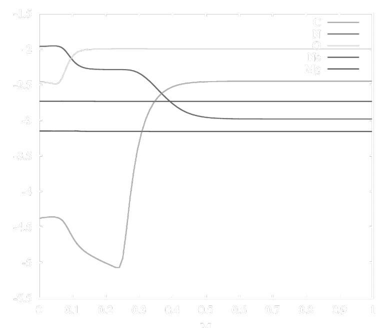
Nuclear Burning In Stars

- All stars burn H to He, e.g. CNO cycle



Internal stellar evolution

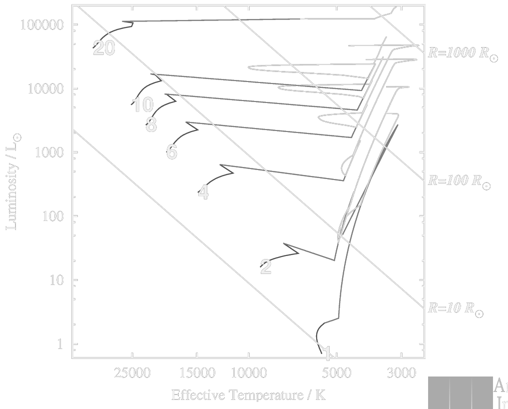
- Composition changes inside a star



$1\,\mathrm{M}_\odot$

TAMS

Mass transfer



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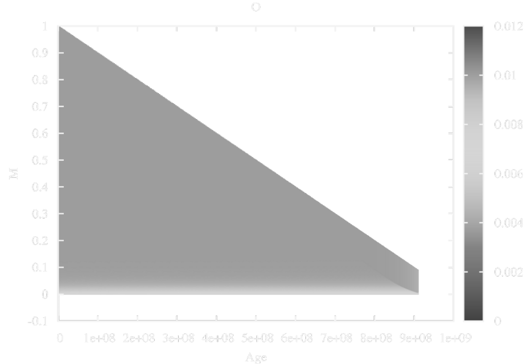
Stripping a solar-mass star



Models made with *Window To The Stars*
http://www.astro.uni-bonn.de/~izzard/window.html

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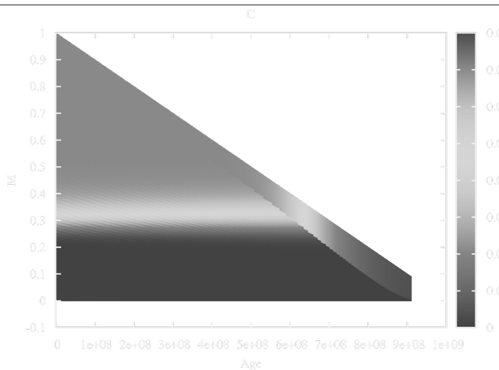
Stripping a solar-mass star



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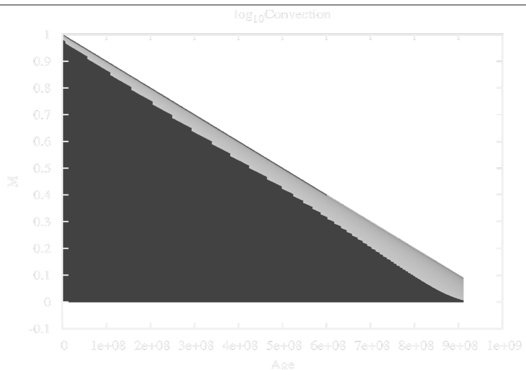

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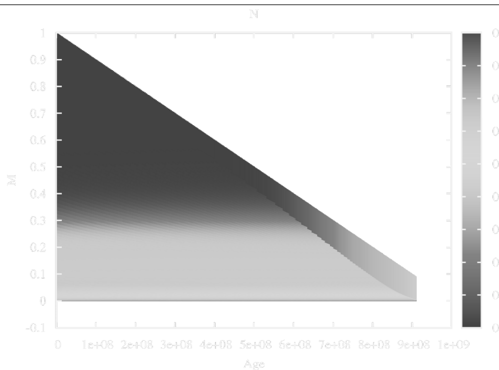

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Algol observations

- Algols have N-enriched mass donors
- Stripping leads to exposed layers

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Observed e.g. LZ Cep

Mahy et al 2011
ArXiv 1106.6162

Table 2. Orbital solution and stellar parameters.

	Primary	Secondary
P [d]	3.070507 (fixed)	
e	0.0 (fixed)	
T_0 [HJD – 2450000]	5032.019±0.002	
$q(M_1/M_2)$	2.53±0.05	
γ [km s^{-1}]	−11.81±0.91	−11.40±1.20
K [km s^{-1}]	88.72±1.02	224.48±2.58
$a \sin i$ [$R_\odot$]	5.38±0.06	13.61±0.16
$M \sin^3 i$ [$M_\odot$]	7.00±0.18	2.77±0.05
rms [km s^{-1}]	2.9198	
T_{eff} [K]	32000±1000	28000±1000
$\log \frac{L}{L_\odot}$	$5.11^{+0.10}_{-0.10}$	$4.69^{+0.10}_{-0.13}$
$\log g$	3.5±0.1	3.1±0.1
M_{spec} [$M_\odot$]	$15.9^{+8.8}_{-6.4}$	$4.1^{+2.4}_{-2.1}$
M_{ev} [$M_\odot$]	$25.3^{+12}_{-4.9}$	$18.0^{+2.4}_{-2.5}$
radius [$R_\odot$]	$11.7^{+2.3}_{-2.7}$	$9.4^{+2.3}_{-2.2}$
He/H	0.1±0.02	0.4±0.1
C/H [× 10^{-4}]	1.0±0.5	0.3±0.2
N/H [× 10^{-4}]	0.85±0.2	12.0±2.0
O/H [× 10^{-4}]	3.0±0.5	0.5±0.3
$V \sin i$ [km s^{-1}]	130±10	80±10
v_{mic} [km s^{-1}]	40±5	44±5
$\dot{M}$ [$10^{-8} M_\odot$ yr^{-1}]	1.0±0.3	–
v_∞ [km s^{-1}]	1800±100	–

Notes. The given errors correspond to 1–σ. The solar abundances for the chemical elements quoted here are He/H = 0.1, C/H = 2.45 × 10^{-4}, N/H = 0.60 × 10^{-4}, O/H = 4.57 × 10^{-4}, respectively.

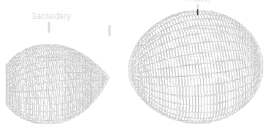
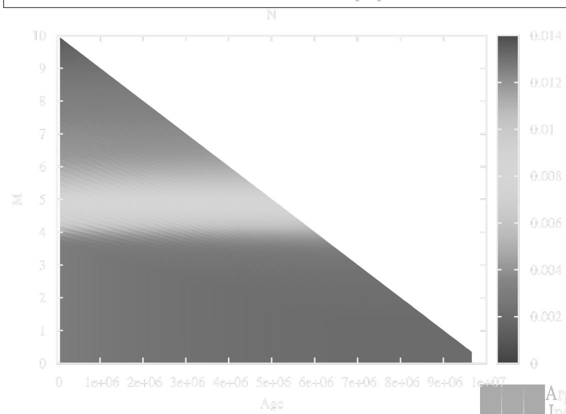


Table 3. Parameters fitted from the Hipparcos light curve.

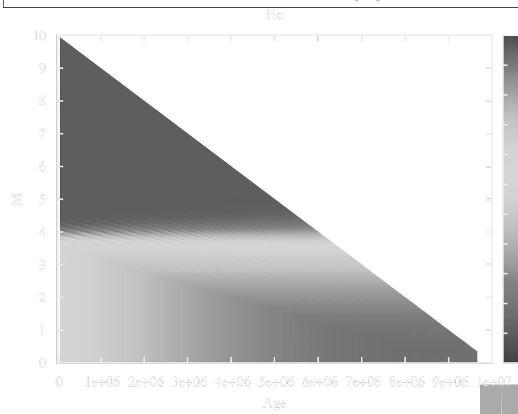
Parameters	Sol 1	Sol 2
i [°]	$50.1^{+2.1}_{-1.5}$	$48.1^{+2.0}_{-0.7}$
q [M_1/M_2]	2.53 (fixed)	2.53 (fixed)
Filling factor primary [%]	$98.0^{+1.2}_{-1.0}$	$94.8^{+2.0}_{-3.0}$
Filling factor secondary [%]	$87.0^{+3.0}_{-5.0}$	$98.6^{+0.3}_{-1.5}$
$T_{eff,p}$ [K]	32000 (fixed)	32000 (fixed)
$T_{eff,s}$ [K]	28000 (fixed)	28000 (fixed)
M_p [$M_\odot$]	$15.5^{+1.0}_{-1.0}$	$16.9^{+1.0}_{-1.0}$
M_s [$M_\odot$]	$6.1^{+1.0}_{-1.0}$	$6.7^{+1.0}_{-1.0}$
$R_{pole,p}$ [$R_\odot$]	$10.5^{+1.2}_{-1.2}$	$10.5^{+1.2}_{-1.2}$
$R_{pole,s}$ [$R_\odot$]	$6.1^{+1.2}_{-1.2}$	$7.1^{+1.2}_{-1.2}$
$R_{equ,p}$ [$R_\odot$]	$13.1^{+1.2}_{-1.2}$	$12.4^{+1.2}_{-1.2}$
$R_{equ,s}$ [$R_\odot$]	$6.9^{+1.2}_{-1.2}$	$9.3^{+1.2}_{-1.2}$

Notes. The index 'p' ('s') refers to the primary (secondary). R_{pole} is the polar radius, and R_{equ} the equatorial radius.

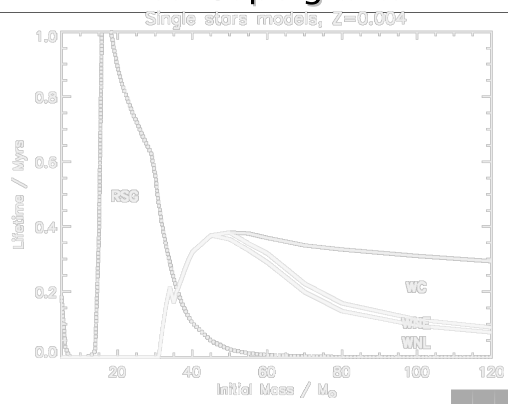
10Msun stripped



10Msun stripped

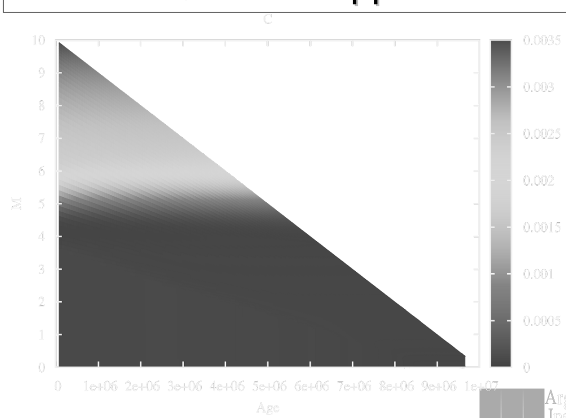


Red Supergiants

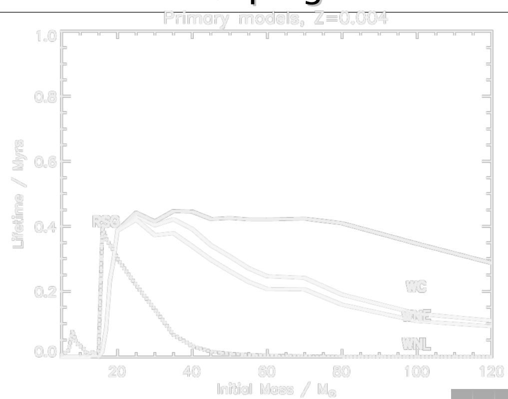


Eldridge, Izzard & Tout 2008, A&A

10Msun stripped

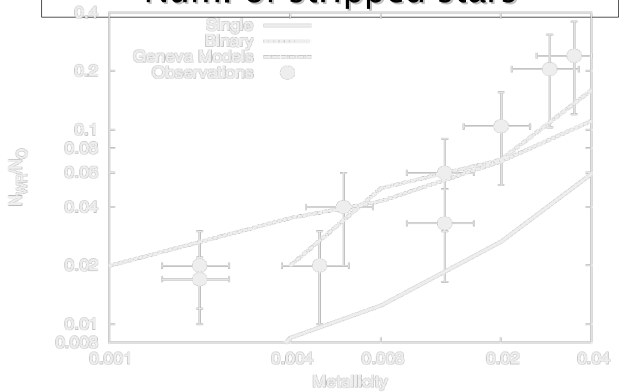


Red Supergiants



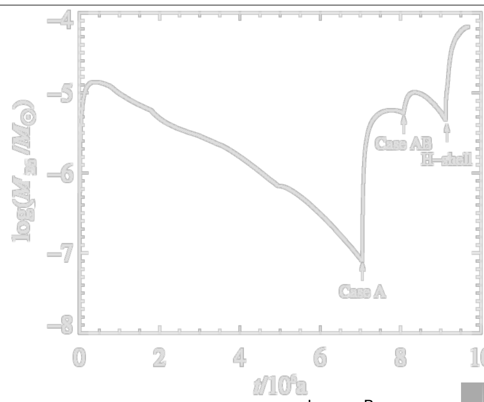
Eldridge, Izzard & Tout 2008, A&A

Num. of stripped stars



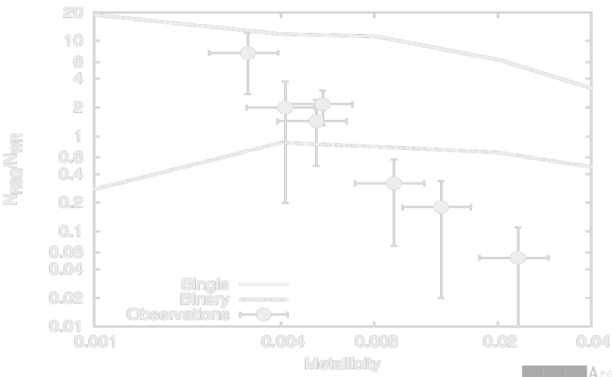
Eldridge, Izzard & Tout 2008, A&A

Effects of Accretion: Al26



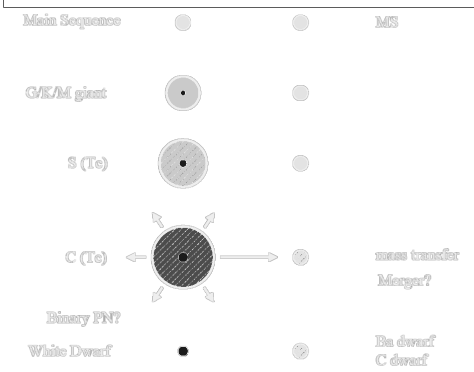
Langer, Braun, Wellstein 1998

Num. of stripped stars

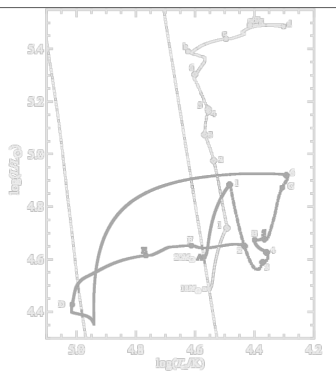


Eldridge, Izzard & Tout 2008, A&A

Wind mass transfer



Effects of Accretion: Al26



Langer, Braun, Wellstein 1998

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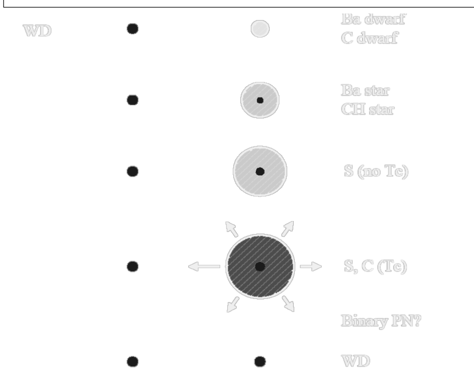
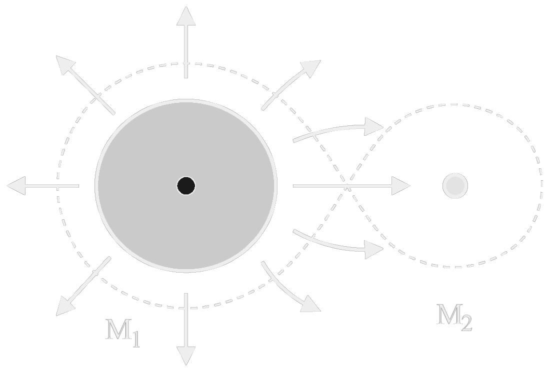
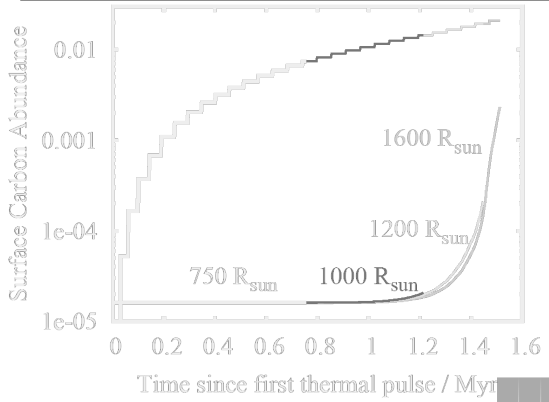


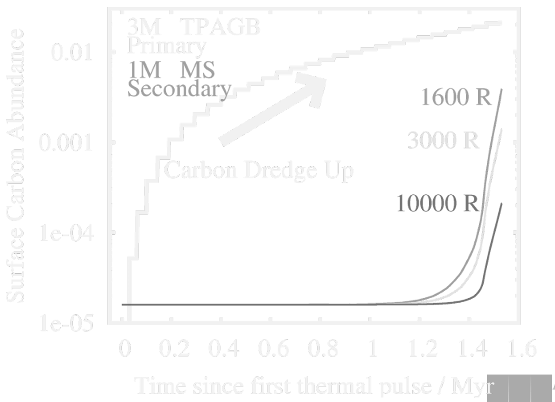
Wind Accretion



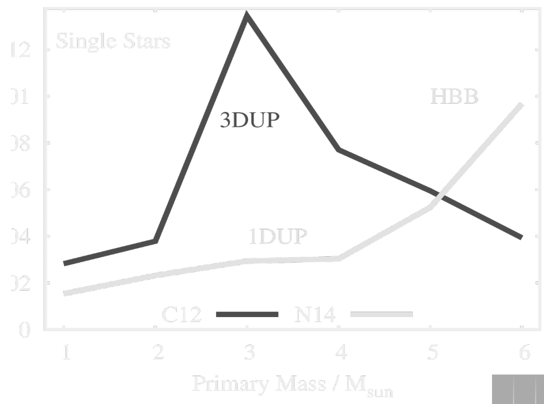
Also RLOF if close enough



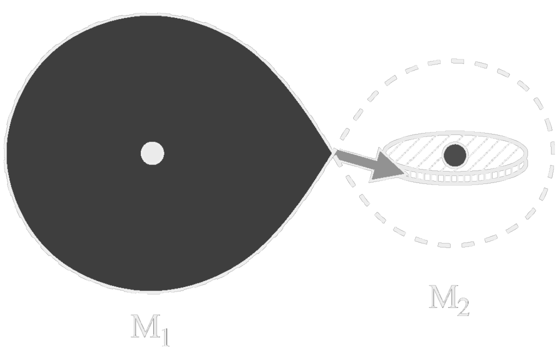
Wider systems: barium/CH stars



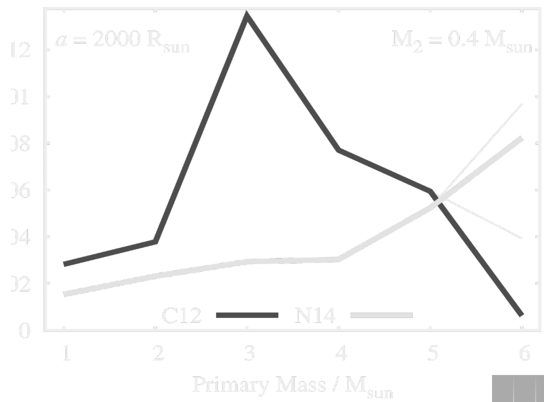
Wider systems: barium/CH stars



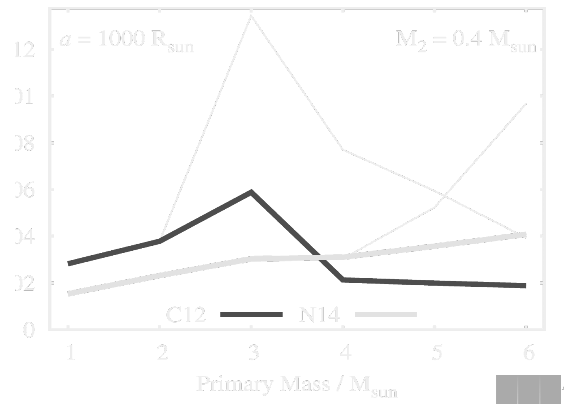
Also RLOF if close enough



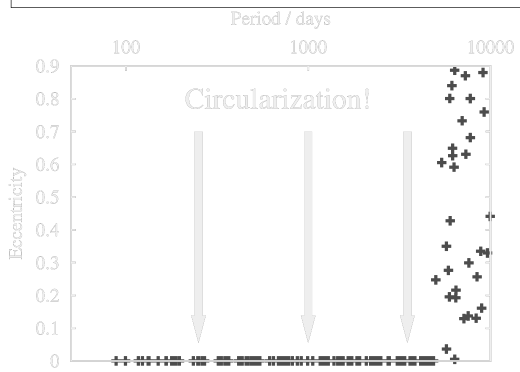
Wider systems: barium/CH stars



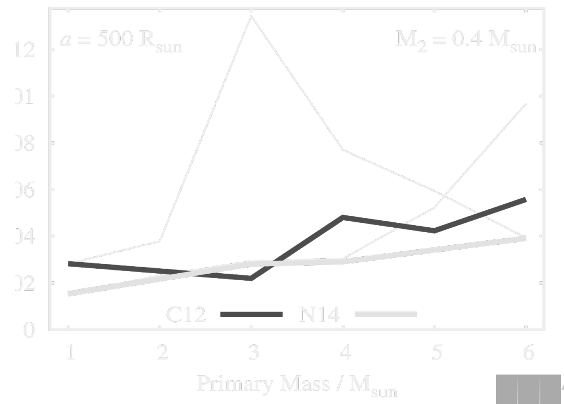
Wider systems: barium/CH stars



Barium Stars and eccentricity



Wider systems: barium/CH stars



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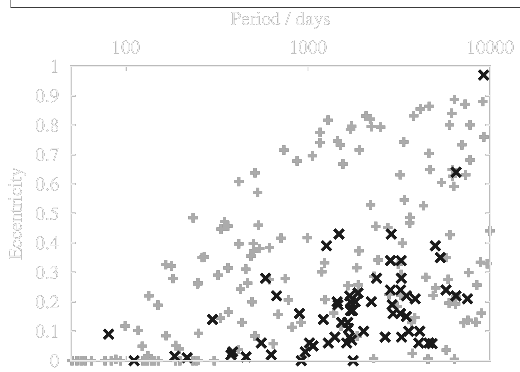
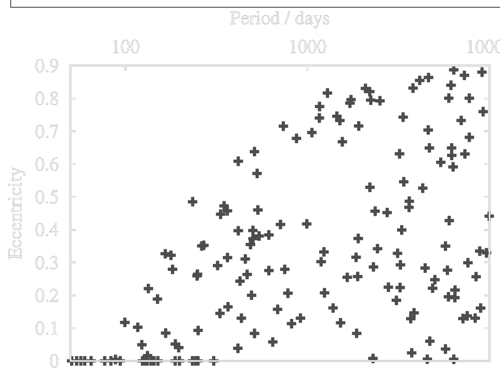


Barium Stars and eccentricity



Thermohaline mixing

- *What happens to material that accretes?*
- *In general it comes from an evolved star i.e. one in which* $\mathrm{H} \rightarrow \mathrm{He}$, $\mathrm{C,N,O} \rightarrow \sim 98\%\,\mathrm{N}$ *etc.*
- *i.e. the molecular weight is larger*

$$\rho = n \times m_{\mathrm{H}} \times \mu$$

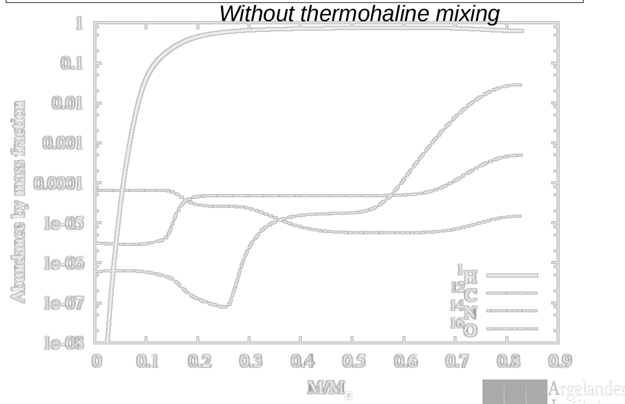
$$\mu = \frac{4}{6X + Y + 2}$$

- *Unstable to thermohaline instability*
- *See e.g.* https://secure.wikimedia.org/wikipedia/en/wiki/Thermohaline

Thermohaline in ink

CEMP star: [C/Fe]=3.25

Without thermohaline mixing



Stancliffe et al 2007 A&A 464,L57

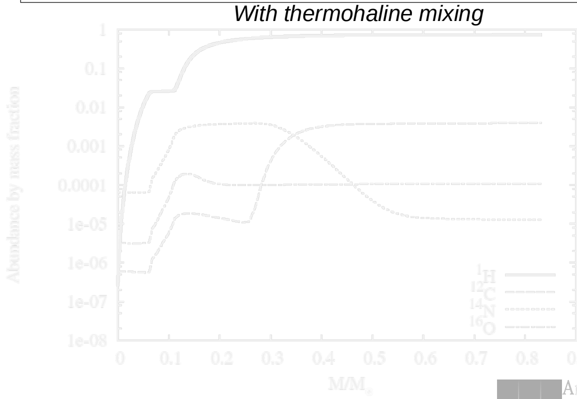
Thermohaline in stars

- Relies on thermal transport so instability occurs on thermal timescale (i.e. fast c.f. $t_{\rm ms}$)
- Kippenhahn et al. 1998: diffusion model

$$D_{\rm th} = \frac{16acT^3 H_{\rm P}}{(\nabla_{\rm ad} - \nabla) c_{\rm P} \rho \kappa} \left| \frac{d\mu}{dr} \right| \frac{1}{\mu}$$

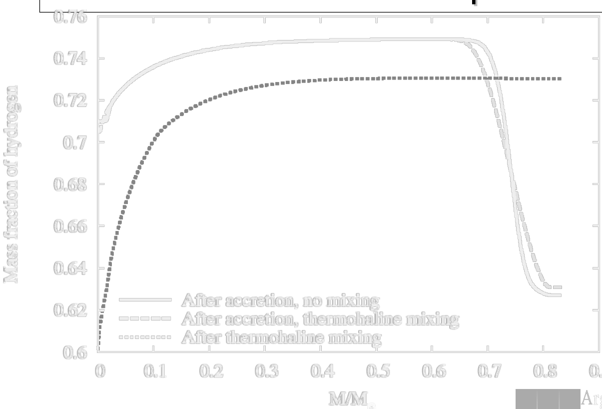
CEMP star: [C/Fe]=2.41

With thermohaline mixing



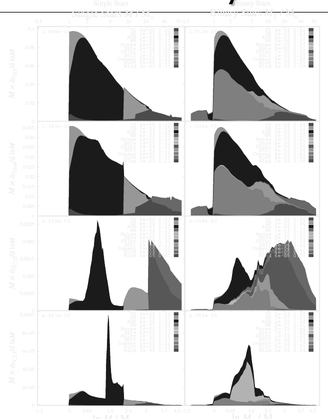
Stancliffe et al 2007 A&A 464,L57

Thermohaline example

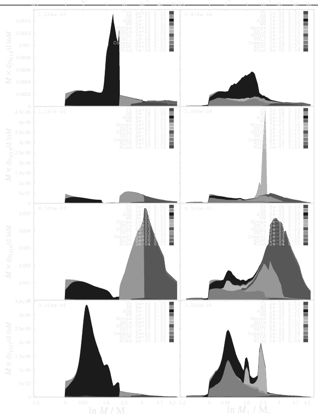


Stancliffe et al 2007 A&A 464,L57

Chemical yields



Chemical yields



Discretisation

- Simplest case: mass conservation

$$dm = 4\pi\rho\, r^2 \times dr$$

- A possible discretisation:

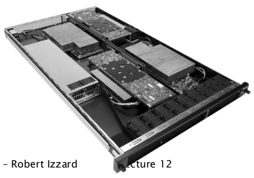
$$M_{i+1} - M_i = \frac{4\pi}{3}\rho_{i+\frac{1}{2}}\left[r_{i+1}^3 - r_i^3\right]$$

- Repeat for other equations/variables

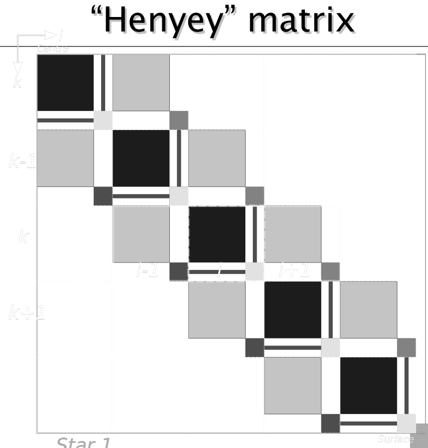
$$T,\ P,\ r \text{ and } \ln f \text{ (degeneracy)}$$

Part 2: Modelling Binary Stars

- Traditional stellar models
- Rapid stellar codes
- Population synthesis
- Parameter space and initial distributions
- Stellar accountancy
- Examples of the power of population synthesis



"Henyey" matrix



Traditional stellar modelling

- Stellar structure equations

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \qquad \frac{dL}{dm} = \epsilon$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \qquad \frac{dT}{dm} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{F}{(4\pi r^2)^2}$$

- Stiff equations
- Solving them is CPU expensive

Detailed code runtimes

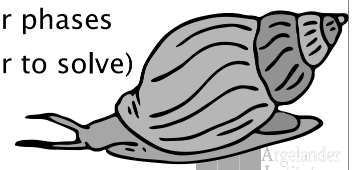
- Say we want *N* timesteps
- These take Δt per timestep
- Total runtime per star
- $t_{\mathrm{CPU}} \sim N\Delta t$
- $\sim \frac{\tau}{\delta t}\Delta t$
- Typically (for an AGB star):

$$\tau \sim 1\,\mathrm{Myr} \qquad \delta t \sim 1\,\mathrm{year} \qquad \Delta t \sim 10\,\mathrm{s}$$

$$t_{\mathrm{CPU}} \sim 10^7\,\mathrm{s}$$

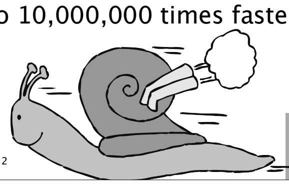
Binary Star Equations

- Twice everything in a single star model
- Binary interaction equations (2, 3, more?)
- Runtime *at least*
- $$t_{\rm CPU}({\rm binary}) \gtrsim 2 \times t_{\rm CPU}({\rm single})$$
- Will be even more in complicated mass-transfer phases
- Bigger matrix (slower to solve)

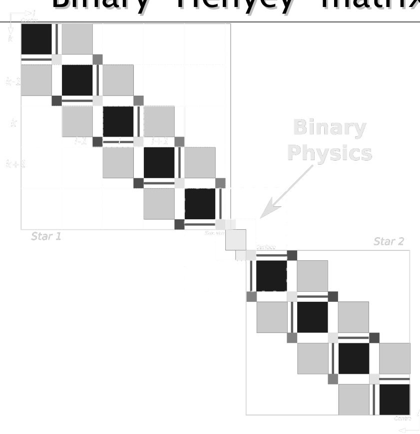


Rapid Stellar Models

- Creating *detailed* stellar models is slow and difficult
- Rapid or synthetic stellar models are faster
- Replace details solver with pre-solved model set:
  - Fitting formulae
  - Or lookup tables
- Sacrifice (usually unwanted) details for speed: up to 10,000,000 times faster.



Binary "Henyey" matrix



Fitting Formulae

- Eggleton, Fitchett, Tout 1989, Hurley et al 2000,2002
- Zero-age main sequence:

$$L_0 = \begin{cases} \frac{1.107M^3+240.7M^9}{1+281.9M^4} & M \leq 1.093 \\ \frac{13990M^5}{M^4+2151M^2+3908M+9536} & M \geq 1.093 \end{cases}$$

$$R_0 = \begin{cases} \frac{0.1148M^{1.25}+0.8604M^{3.25}}{0.04651+M^2} & M \leq 1.334 \\ \frac{1.968M^{2.887}-0.7388M^{1.679}}{1.821M^{2.337}-1} & M \geq 1.334 \end{cases}$$

An aside: dimensions of rotating stars

- Can we treat stars as essentially *single stars*?
- Polar radius is approx const.

Variations in the polar radius as a function of the ratio $\omega = \Omega/\Omega_{\rm crit}$, normalized to the non-rotating value, for various masses at standard metallicity.

Ekstrom et al 2008 A&A 478, 467

Fitting Formulae

- Time evolution function of $\tau = t/t_{\rm MS}$

$$t_{\rm MS} = \frac{2550 + 669M^{2.5} + M^{4.5}}{0.0327M^{1.5} + 0.346M^{4.5}}.$$

- Then

$$\log_{10} L = \log_{10} L_0 + \alpha\tau_{\rm MS} + \beta\tau_{\rm MS}^2$$
$$\log_{10} R = \log_{10} R_0 + \alpha'\tau_{\rm MS} + \beta'\tau_{\rm MS} + \gamma'\tau_{\rm MS}^3$$

Fitting formulae

$$\alpha = \begin{cases} 0.2594 + 0.1348 \log_{10} M & M \le 1.334 \\ 0.09209 + 0.05934 \log_{10} M & M > 1.334 \end{cases}$$

$$\beta = \begin{cases} 0.144 - 0.833 \log_{10} M & M \le 1.334 \\ 0.3756 \log_{10} M - 0.1744 (\log_{10} M)^2 & M > 1.334 \end{cases}$$

$$\alpha' = \begin{cases} 0 & M \le 1.334 \\ 0.1509 + 0.1709 \log_{10} M & M > 1.334 \end{cases}$$

$$\beta' = \begin{cases} 0.2226 \log_{10} M & M \le 1.334 \\ -0.4805 \log_{10} M & M > 1.334 \end{cases}$$

$$\gamma' = \begin{cases} 0.1151 & M \le 1.334 \\ 0.5083 \log_{10} M & M > 1.334 \end{cases} .$$

Even more complicated formulae apply for later phases of evolution!
But computers *do not care ...*

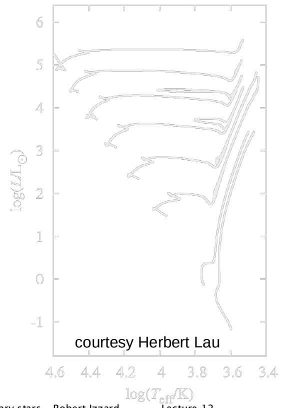
Population Synthesis

The process of combining stellar models into a stellar population upon which meaningful statistical analysis can be performed and compared to observations to better constrain the underlying physics.

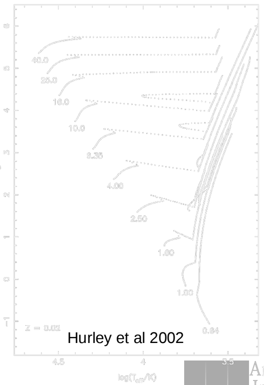
1. Make your stellar models
2. Weight these according to mass, separation, time etc.
3. Extract simulated value(s)-compare
4. Determine the “real-life” distribution from obs.
5. Compare the two, see what's wrong
6. Refine your stellar models
7. Return to step 1 until you are happy

(or funding runs out)

Real vs Synthetic HRD



courtesy Herbert Lau


Hurley et al 2002

The Parameter Space Problem

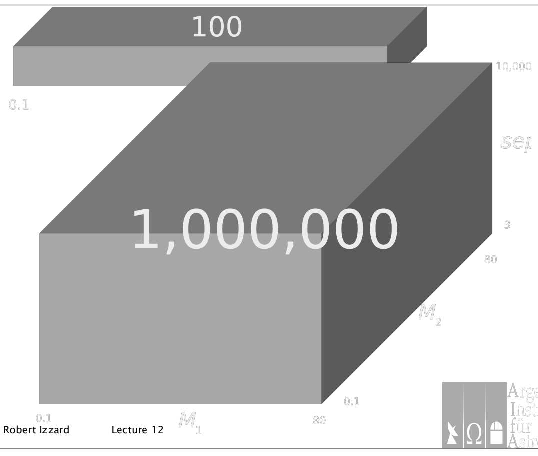
- To make a single star population, one parameter only: Mass M_1
- Runtime is $\sim N \times \Delta t$
- Binaries many parameters :
  - Primary mass M_1
  - Secondary mass M_2
  - Sep/Period a or P
  - Maybe more e.g. e
- Runtime $\sim N^3 \times \Delta t$

Pros and Cons

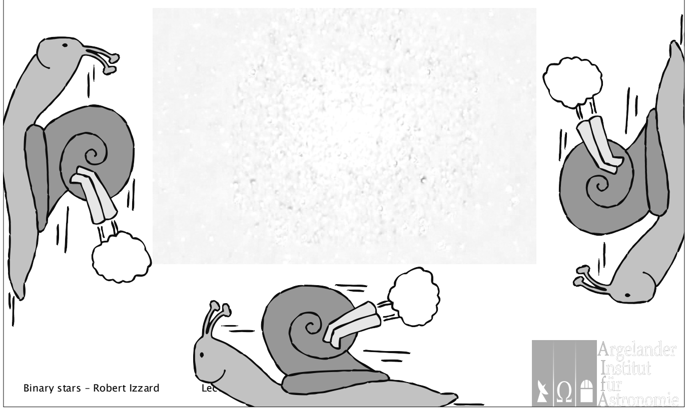
- Pros
  - Faster to compute
  - Stable
- Cons
  - Fixed input physics (but could use tables!)
  - Discard of potentially useful information
  - Off-grid treatment
  - Fitting errors (<5%)

$$\log_{10} L = \log_{10} L_0 + \alpha \tau_{\rm MS} + \beta \tau_{\rm MS}^2$$
$$\log_{10} R = \log_{10} R_0 + \alpha' \tau_{\rm MS} + \beta' \tau_{\rm MS} + \gamma' \tau_{\rm MS}^3$$

Parameter Spaces



Popsyn + rapid code



Discretising Parameter Space

- *Binary Stars*

$$\delta \ln x \quad = \quad \frac{\ln x_{\max} - \ln x_{\min}}{n_x}$$

where x is $M_1,\, M_2,\, a(,\, P, e\ldots)$

- *Each star has a probability of existence*

$$\delta p_i \quad = \quad \Psi_i(M_1,\, M_2,\, a)\,\delta V$$

- *Where* Ψ *is the initial distribution function*

Discretising Parameter Space

- *Single Stars*

$$\delta \ln M \quad = \quad \frac{\ln M_{\max} - \ln M_{\min}}{n}$$

- *Each star has a probability of existence*

$$\delta p_i \quad = \quad \psi(M_i)\,\delta \ln M$$

- *Where* ψ *is the initial mass function*

$$\Sigma_i \delta p_i = 1$$

Initial Distribution Function

$$\Psi_i \quad = \quad \psi\left(M_{1i}\right)\phi\left(M_{2i}/M_{1i}\right)\chi\left(a_i\right)$$

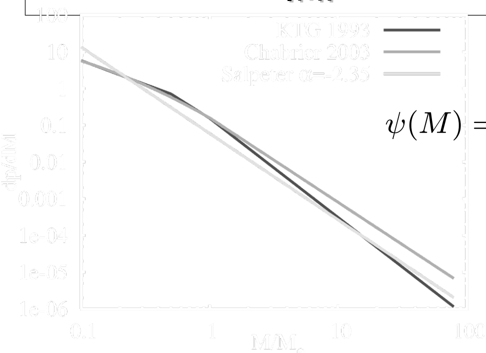
$$\begin{aligned} \psi(M_1) &= \psi(M) \\ \phi\left(q = \frac{M_1}{M_2}\right) &= \text{constant} \\ \chi(a) &\propto a^{-1} \\ \chi(\ln a) &= \text{constant}\,. \end{aligned}$$

$$\delta p_i \quad = \quad \Psi_i \delta V_i$$

$$\delta V \quad = \quad \delta \ln M_1 \delta \ln M_2 \delta \ln a$$

$$\Sigma_i \delta p_i = 1$$

IMF



$$\psi(M) = \frac{dN_{stars}}{dM}$$

Salpeter IMF $\psi \propto M^{-2.35}$

Stellar accounts

- Define

$$\begin{aligned} \delta(\text{phase}) &= 1 \quad \text{during the phase,} \\ &= 0 \quad \text{otherwise}\,. \end{aligned}$$

- Time a star spends in a phase of interest

$$\Delta t_i = \sum_{t=t_{\min}}^{t_{\max}} \delta(\text{phase at } t)_i\, \delta t$$

Stellar accounts

- The number of stars in the phase is

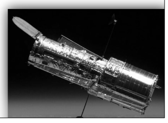
$$\begin{aligned} \text{count} &= \sum_i S\delta p_i \Delta t_i \\ &= \sum_i S\delta p_i \sum_{t_{\min}}^{t_{\max}} \delta(\text{phase})_i \, \delta t \end{aligned}$$

where *S* is the star formation rate

- In general we have to convolve a birth function with a star formation rate function
-

Compare to Observations

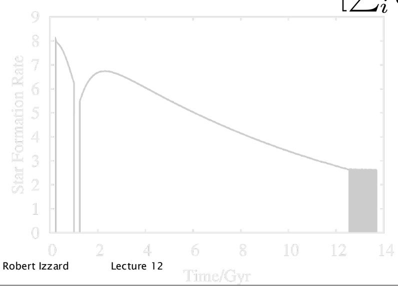
- Statistics!
  - Boring (but not for everyone!)
  - Necessary e.g. χ^2,KS tests
  - Key to good science
- Beware observational selection effects
  - Often very hard to model
  - Data combined from multiple surveys might be impossible to model!
  - Sometimes whole papers are wrong because they neglect this! (not deliberately)



Stellar accounts

- Simple case : *S* = constant
- Divide counts to get ratios : *S* drops out

$$\text{ratio} = \frac{[\sum_i \delta p_i \Delta t_i]_1}{[\sum_i \delta p_i \Delta t_i]_2} .$$



Galactic SFR
Chiappini et al 1997

A rapid code: *binary_c*

- My code, my lectures, so …
- Based on *SSE/BSE* of EFT89, Hurley et al 2000, 2002 (e.g. see prev. eqs)
- Has fitting functions for stellar evolution
- +orbital algorithm: RLOF, Wind, Tides
- Common env., Novae, SNe Ia, Mergers etc.
- Online
- **http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cgi**

Stellar accounts

- The number of stars in the phase is

$$\Sigma_i S \delta p_i \Delta t_i$$

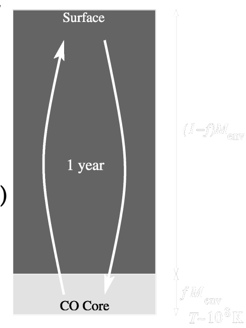
where *S* is the star formation rate

- In general we have to convolve a birth function with a star formation rate function

$$\sum_{t'_{\min}}^{t'_{\max}} \sum_i S(t) \delta p_i \delta(\text{phase at } t')_i \, \delta t'$$

binary_c/nucsyn

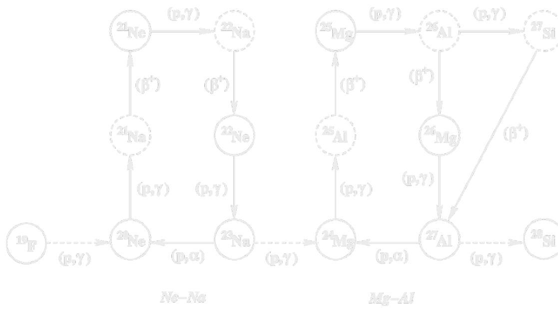
- Added *nucleosynthesis* to *binary_c*
- First and second dredge up
- TPAGB based on Karakas' models:
  - Third dredge up
  - Hot-bottom burning (CNO, NeNa, MgAl)
  - S-process (Torino group)
- SN II/Ibc yields, novae
- Thermohaline mixing
- Physics updates over last few years



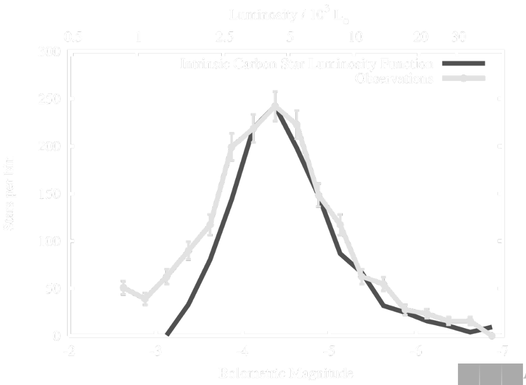
Some examples of binary_c

- *Remember to try it yourself!*
- http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cgi

Nuclear Burning Rates



Low-L Carbon Stars

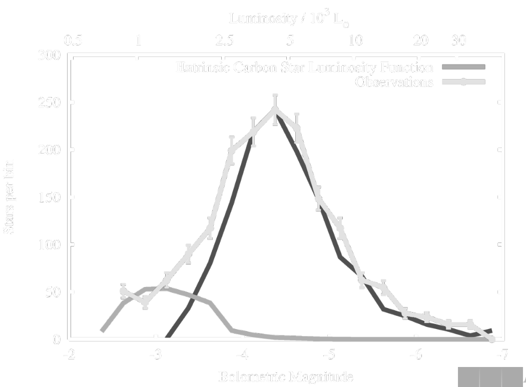


Izzard and Tout 2004

Nuclear Burning Rates

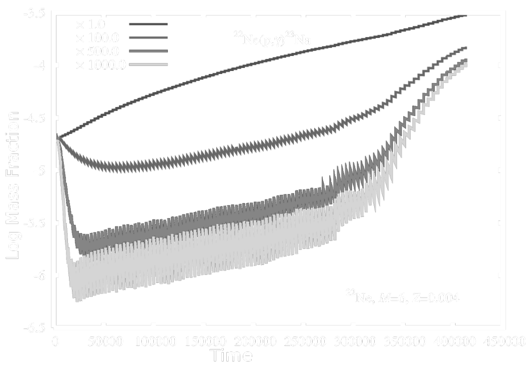
Rate			Source
^{20}Ne$(p,\gamma)^{21}$Na$(\beta^+)^{21}$Ne	-50%	$+50\%$	NACRE
^{21}Ne$(p,\gamma)^{22}$Na$(\beta^+)^{22}$Ne	-20%	$+20\%$	Iliadis et al. 2001
^{22}Ne$(p,\gamma)^{23}$Na	-50%	$\times 2000$	Hale et al. 2001
^{23}Na$(p,\alpha)^{20}$Ne	-30%	$+30\%$	Rowland et al. 2004
^{23}Na$(p,\gamma)^{24}$Mg	$/40$	$\times 10$	Rowland et al. 2004
^{24}Mg$(p,\gamma)^{25}$Al$(\beta^+)^{25}$Mg	-17%	$+20\%$	Powell et al. 1999
^{25}Mg$(p,\gamma)^{26}$Al$(\beta^+)^{26}$Mg	-50%	$\times 1.5$	Iliadis et al. 2001
^{26}Mg$(p,\gamma)^{27}$Al	$/4$	$\times 10$	Iliadis et al. 2001
^{26}Mg$(p,\gamma)^{27}$Al	-25%	$\times 3$	Iliadis et al. 2001
^{26}Al$(p,\gamma)^{27}$Si	$/2$	$\times 600$	Iliadis et al. 2001

Low-L Carbon Stars



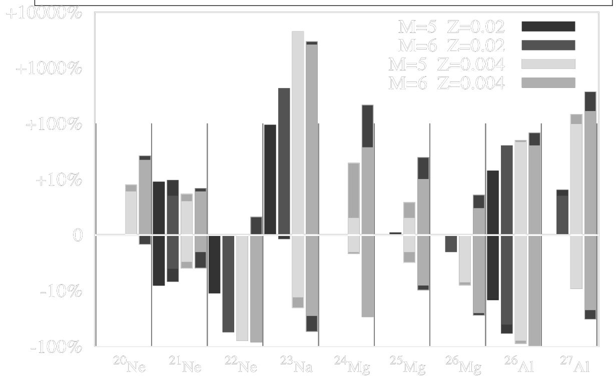
Izzard and Tout 2004

Nuclear Burning Rates



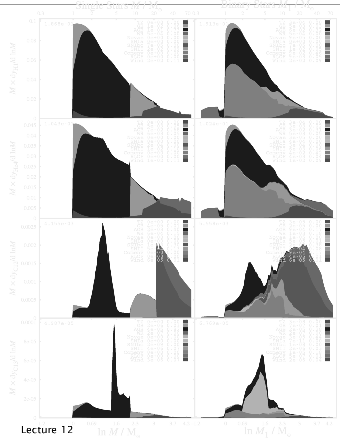
Izzard et al. 2007

Nuclear Burning Rates



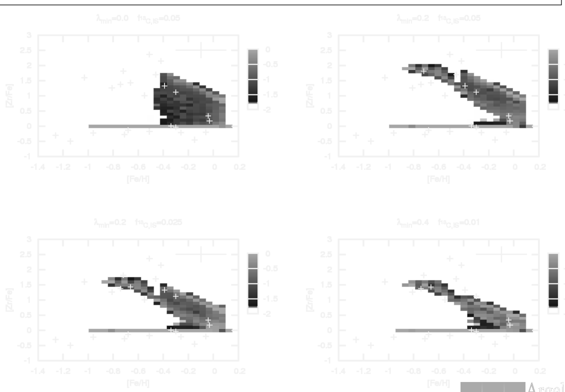
Izzard et al. 2007

Ejecta from stellar populations



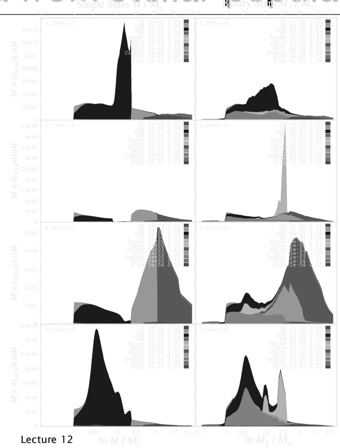
Izzard PhD!

s-process in post-AGB



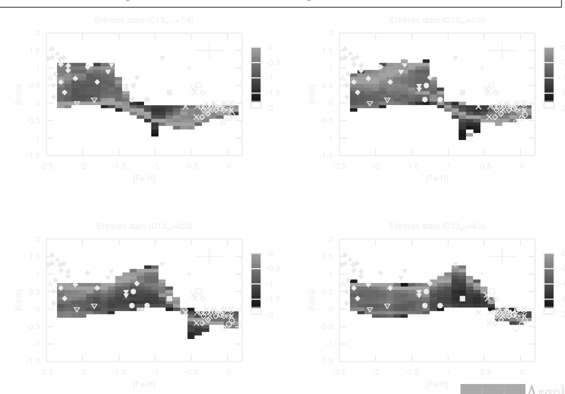
Bonacic et al. 2007

Ejecta from stellar populations



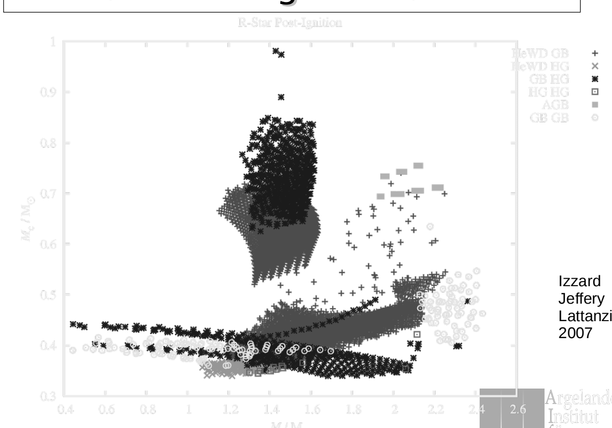
Izzard PhD!

s-process in post-AGB



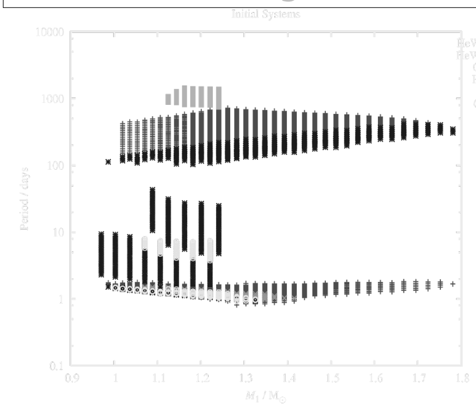
Bonacic et al. 2007

Stellar Mergers: R Stars



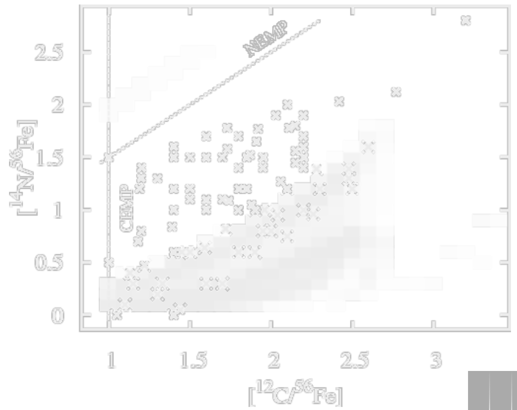
Izzard
Jeffery
Lattanzio
2007

Stellar Mergers: R Stars

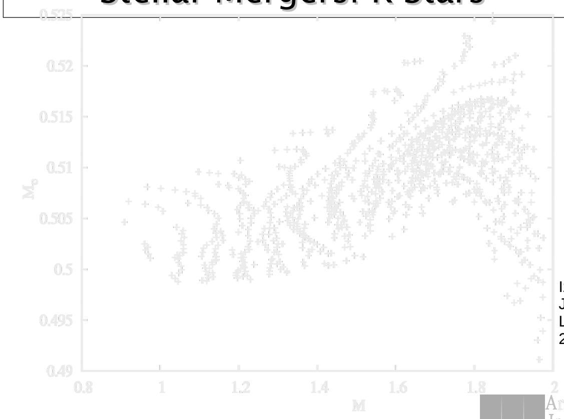


Izzard
Jeffery
Lattanzio
2007

CEMP stars

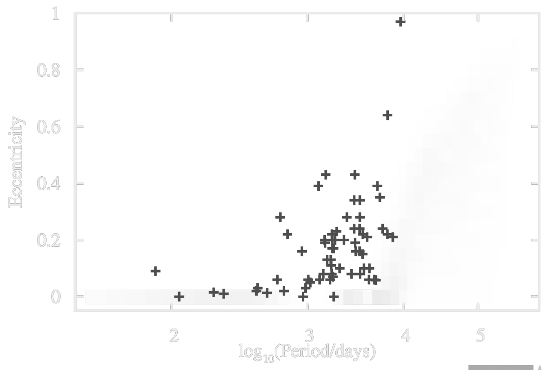


Stellar Mergers: R Stars



Izzard
Jeffery
Lattanzio
2007

Barium Stars

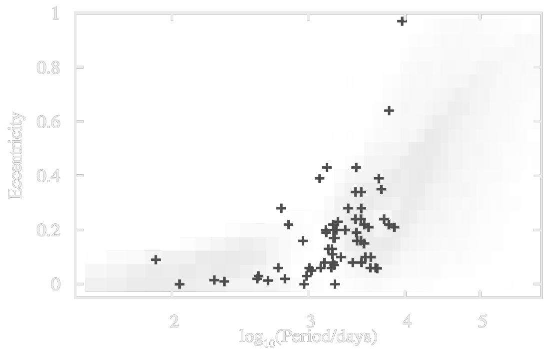


Izzard et al 2009

CEMP stars

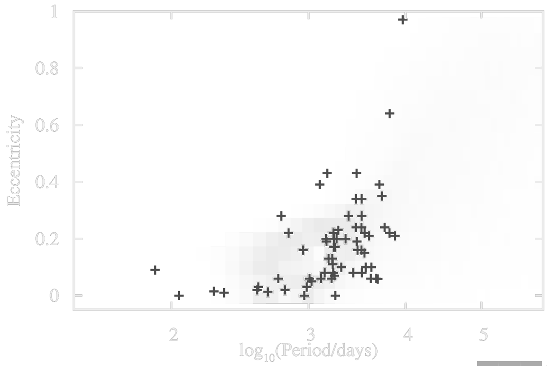
Izzard et al 2009

Barium Stars



Izzard et al 2009

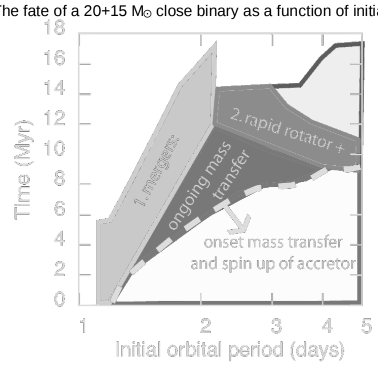
Barium Stars



Izzard et al 2009

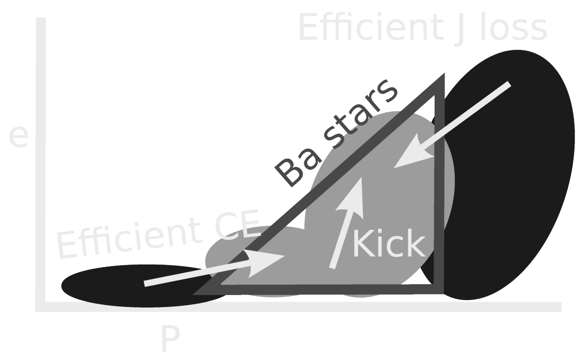
Massive Stars

The fate of a 20+15 $M_{\odot}$ close binary as a function of initial period.



De Mink et al. 2010/11

Barium Stars



Izzard et al 2009

The end!

- Exam:

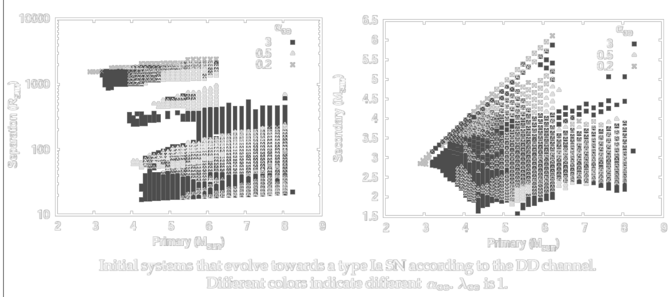
  Tuesday 17th July

  10.00-11.30am

  Herbert Lau will be supervising you.

- Good luck! Thanks for coming :)

Ia Supernovae



Joke Claeys (work in progress!)