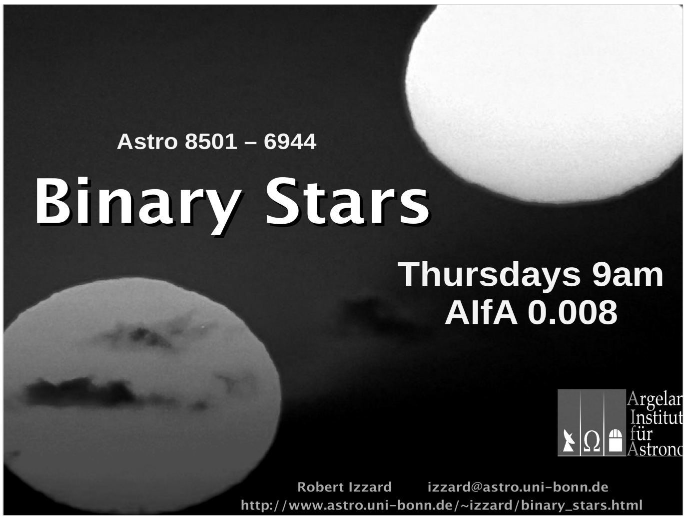
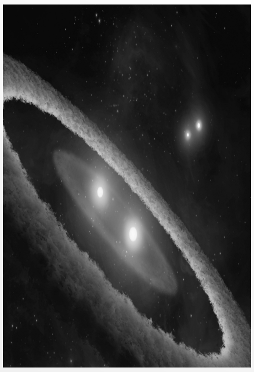


Class 1: An Introduction

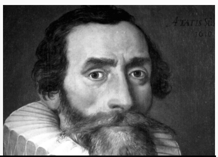
- A bit of history
- Famous binary stars
- Bright stars and binaries
- Types of binaries
- Basic nomenclature
- Resources at your disposal

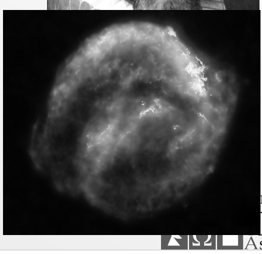


Kepler's Laws

- German!
- 1571-1630
- Worked with

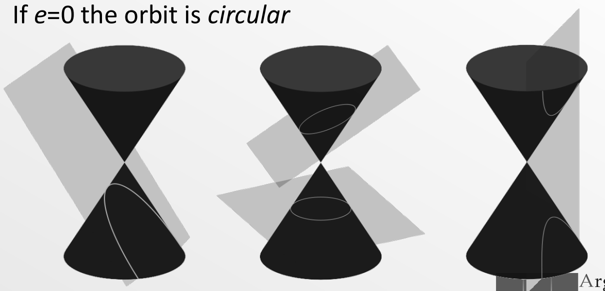
  Tycho Brahe



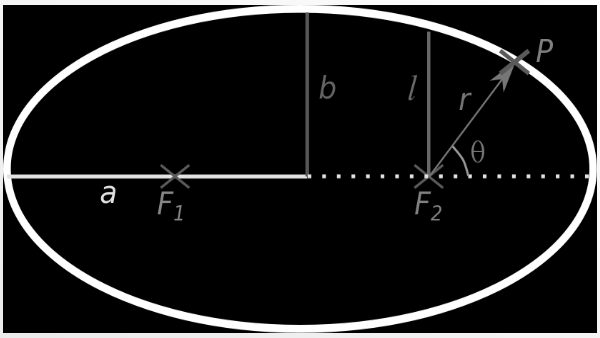


Kepler's First Law

- The orbits of binary stars are *conic sections*
- *Bound* orbits are ellipses
- If *e*=0 the orbit is *circular*

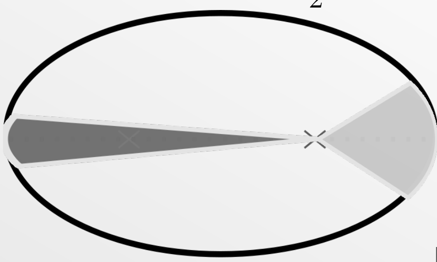


Elliptical Motion



Kepler's Second Law

- The line connecting the two stars sweeps out *equal areas* in *equal times* $\frac{1}{2}r^2\dot{\theta} = \frac{\pi a^2\sqrt{1-e^2}}{P}$



Kepler's Third Law

- Period and separation are related by

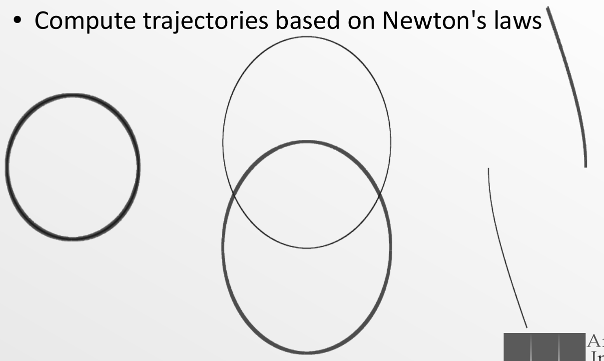
$$P^2 \propto a^3$$

- Independent of eccentricity
- Define mean angular velocity

$$\omega = \frac{P}{2\pi}$$

Numerical Integration of Newton

- Compute trajectories based on Newton's laws



Kepler's Laws

- Bound Orbits are ellipses
- Equal areas swept in equal times
- $P^2 \propto a^3$
- All consequences of Newton's law
- $$F = \frac{GM_1M_2}{r^2}$$

Angular Momentum

- Basic definition

$$\mathbf{J} = M_1\mathbf{r}_1 \times \dot{\mathbf{r}}_1 + M_2\mathbf{r}_2 \times \dot{\mathbf{r}}_2$$

- Is conserved!

$$\dot{\mathbf{J}} = \mathbf{0}$$

Newton's Laws

- Stars are point masses
- Position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in CoM frame
- Define $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$
- Then the forces on the stars are

$$M_1\ddot{\mathbf{r}}_1 = -G\frac{M_1M_2}{r^3}\mathbf{r}_1$$

$$M_2\ddot{\mathbf{r}}_2 = -G\frac{M_1M_2}{r^3}\mathbf{r}_2$$

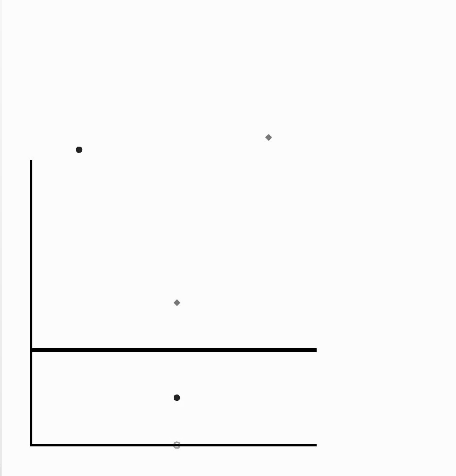
Energy Conservation

- Energy = Kinetic Energy + Potential Energy
- $\dot{E} = 0$ is a consequence of Newton's laws

$$E = \tfrac{1}{2}M_1\left|\dot{\mathbf{r}}_1\right|^2 + \tfrac{1}{2}M_2\left|\dot{\mathbf{r}}_2\right|^2 - \frac{GM_1M_2}{r}$$

$$E = \frac{1}{2}\mu\dot{\mathbf{r}}\cdot\dot{\mathbf{r}} - \frac{GM\mu}{r}$$

$$\dot{E} = 0$$



$$e = 0$$

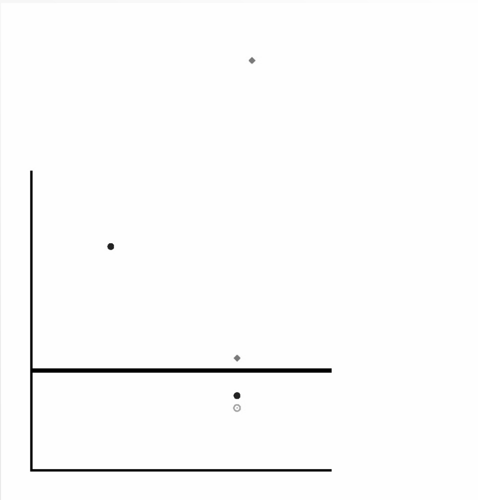
Another invariant

- Laplace-Runge-Lenz vector
- Related to eccentricity vector $\mathbf{e}$

$$GM\mathbf{e} = \dot{\mathbf{r}} \times \mathbf{h} - \frac{GM}{r}\mathbf{r}$$

- Can use this to show

$$E = -\frac{GM_1 M_2}{2a}$$



$$e = 0.5$$

Area of the ellipse

- Hence Kepler's third law

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM}$$

$$e > 1$$

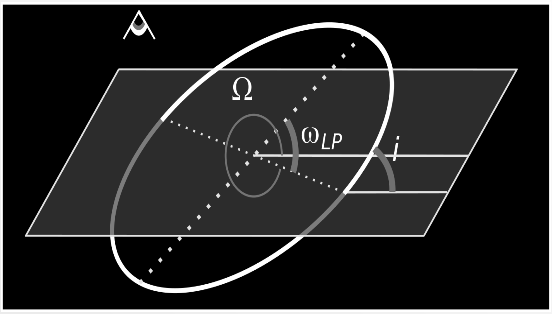
More on orbital elements

Intrinsic properties of the binary:

- Period *P*
- Semi-major axis *a*
  - *P* and *a* : mass of the binary *M* (Kepler 3)
- Eccentricity *e*
- *Reminder:* Periastron = closest approach
- Apastron = furthest approach

More on orbital elements

•



Spectroscopic Binaries

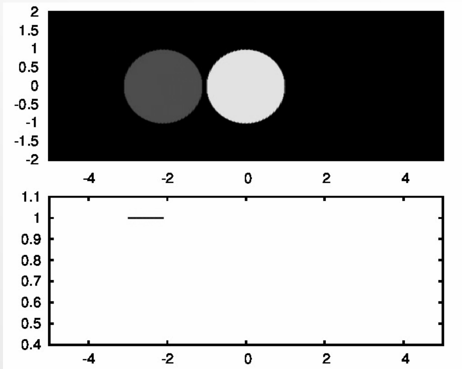
- e.g. CORAVEL 10km/s accuracy, planets 1km/s
- Measure projected velocity: $v \sin i = K$
- Hence the *mass function* gives a lower limit on the stellar mass

$$F_1 = \frac{P}{2\pi G} K_1^3 = \frac{M_2^3 \sin^3 i}{M^2}$$

Extrinsic Properties

- Inclination *i* ... 90 degrees for eclipses
- Ω angle between nodes and a fixed direction
- ω longitude of periastron
- *T* time of periastron passage

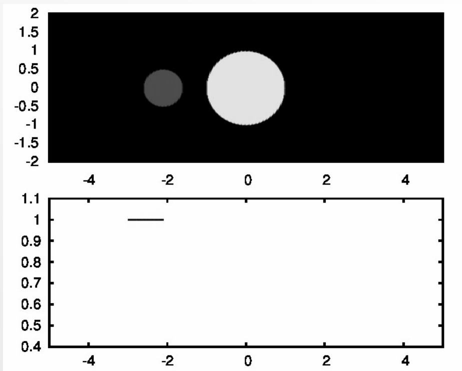
Eclipse shape



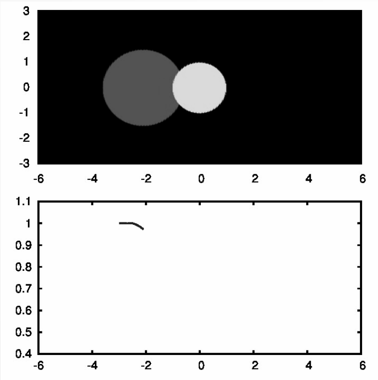
Visual Binaries

- 0.01”, closer for speckle interferometry
- Find *i*, Ω, *e*, ω, measure *P* and *T* with time
- Parallax gives d hence a
- Kepler's law gives M_1 and M_2

Eclipse shape



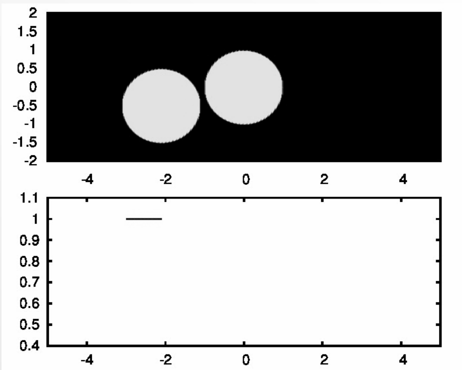
Eclipse shape



Fundamental stellar parameters

- From $K1/2$ we get M_1/M_2 and lower limits $f(M_{1,2})$
- With i we get M_1, M_2 and a
- Eclipse data with distance d gives $R_{1,2}$
- Spectrum gives T_{eff} to get L or get L from *colour* with a bolometric correction
- See e.g. Andersen (1991), Hilditch chapter 6

Eclipse shape gives *i*



Next time

- Close binary stars: *Interacting*
- Tides
- Roche geometry
- Lagrange points
- Introduction to mass transfer?

Eclipse shape gives *i*