Binary Stars - astro8501 - 6944

Problem Sheet 3

1. Show that the rate of change of energy in a binary system in which the spins are aligned can be written

$$\dot{E} \;=\; I\dot{\Omega}\,(\Omega - \omega)$$

where Ω is the angular frequency (spin rate) of the star and ω the orbit. Hence find an expression for $\ddot{E}$ and comment on the stability of the solution in terms of the moments of inertia of the orbit and the star.

2. Estimate the height of tidal bulges in the ocean of Earth due to the Moon.

3. Convection is an efficient means for energy dissipation due to tides. Show that the viscosity ν due to convective motions is approximately $l^2/\tau_{\rm conv}$ where l is the size of the convective zone and $\tau_{\rm conv}$ is the convective turnover time. Can you estimate $\tau_{\rm conv}$ for the Sun?

4. For a(n almost) fully convective typical Galactic AGB star, estimate the tidal dissipation timescale due to convection.

5. *The Darwin instability.*
   Who was Darwin?
   The angular momentum of a sphere rotating with solid-body rotation at angular frequency ω can be written $J = I\omega$ where $I = kMR^2$ is the moment of inertia. Derive I and hence k for a uniform-density sphere. Explain *qualitatively* what you expect k to be for the Earth, Jupiter and the Sun.
   Consider the X-ray binary Cen X-3 with $M_1 = 20\,{\rm M}_\odot$, $R = 12\,{\rm R}_\odot$, $M_2 = 1.4\,{\rm M}_\odot$ and $P_{\rm orb} = 2\,$days. What type of star is the secondary? What is its radius (approximately)? What is the moment of inertia of the orbit and the primary star? Is the system Darwin unstable?
   Now consider planets with a mass $\sim {\rm M}_{♃}$ in close orbits with solar-like stars e.g. $P = 4\,$days (note that ♃ is the symbol for Jupiter). Are these systems stable? Would you be worried if you lived on the planet?
   Consider a contact binary system with $M_1 = 2\,{\rm M}_\odot$, $R_1 = 2\,{\rm R}_\odot$ and $M_2 = 0.5\,{\rm M}_\odot$, $R_2 = 0.5\,{\rm R}_\odot$. Assuming both stars have $k = 0.08$, what is the period below which the Darwin instability is activated?

Questions, problems, errors? Contact Rob Izzard by email: izzard@astro.uni-bonn.de