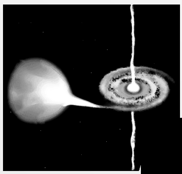
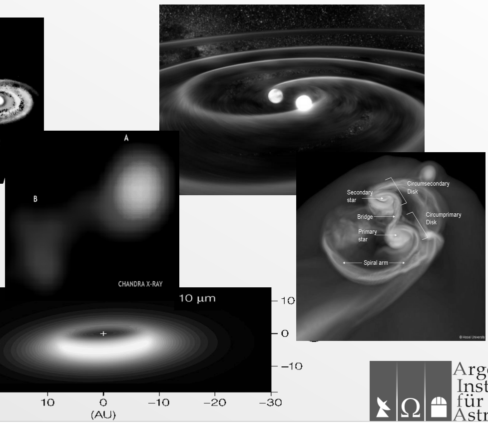
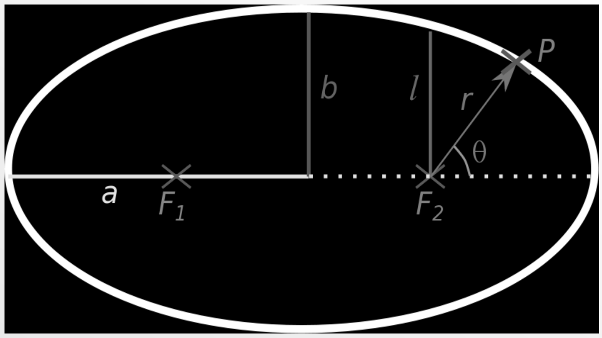
Binary Stars – Lecture 3



Astro 8501
6944



Elliptical Motion



Kepler's Laws

- Bound Orbits are ellipses
- Equal areas swept in equal times

$$P^2 \propto a^3$$

$$\dot{\mathbf{J}} = \mathbf{0} \quad \dot{E} = 0$$

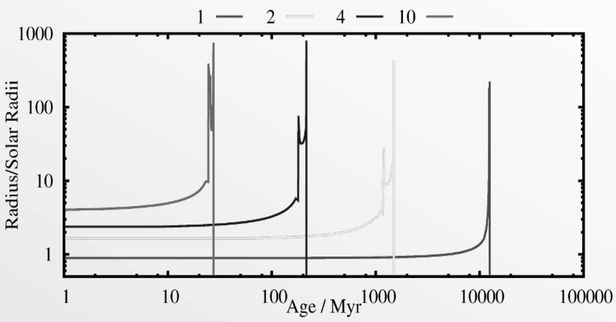
Interacting Binary Stars

- What does *interacting* mean?
  - Gravitational? Material?
- In general interaction occurs when

$$R \rightarrow a$$

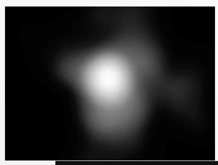
- Which stars and when?

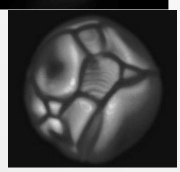
Stellar Evolution



Which stars interact?

- Stars reach radii around $1000\,\mathrm{R}_\odot$ e.g. Betelgeuse
- For $M_1 = M_2 = 1\,\mathrm{M}_\odot$
- And $a = 1000\,\mathrm{R}_\odot \approx 5\,\mathrm{AU}$
- Kepler $\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM}$
- $P \sim 7\,\mathrm{years}$
  - Anything closer interacts strongly!
- Stellar winds as well... (later!)

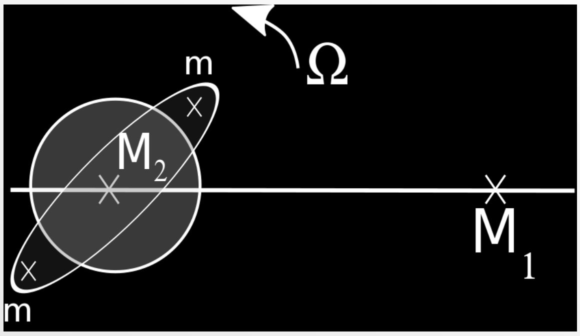




Gravitational interaction

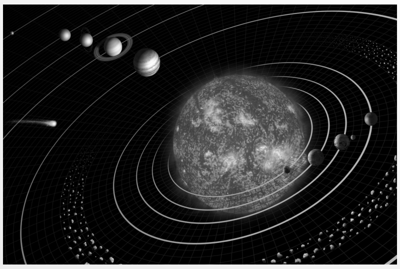
- Gravitational field due to a single star $\phi = \frac{-GM}{r}$
- Companion will feel pull
- Leads to *distortion* known as *tides*
- Familiar ocean tides in Earth-Moon binary system
- Energy Dissipation
- Time lag : Tidal Torque;
  - angular momentum exchange

Tidal System



Tidal Energy Minimum (Orbit)

- Energy minimum when $\dot{E} = 0$
- Equivalent to $e = 0$
- i.e. *circular orbits*



- Example system:
- Sol's planets!
- Most close binaries

Tidal Energy Minimum (Spin)

- Energy minimum when $\dot{E} = 0$
- Equivalent to $\omega = \Omega$
- i.e. *synchronous rotation*

- Example system:
- Earth-Moon
- *Moon* is synchronous
- *Earth* is not (but getting there)

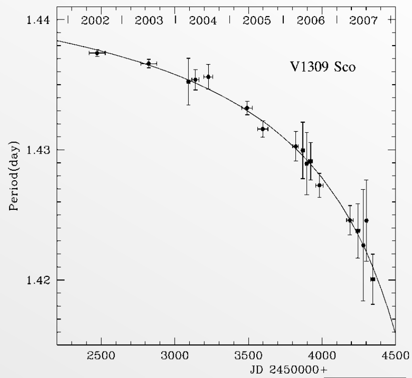
Darwin Instability

- Conservation of angular momentum
- But what if there is not enough angular momentum?
- Momentum from orbit transferred to the star
- Orbit can become unstable...
- Many close planets, contact binaries!
- Why do they not merge immediately?
  - Timescales! Not mentioned yet...

Darwin Instability Seen!

- V1309 Sco
- Tylenda et al.

2011 A&A 528,114



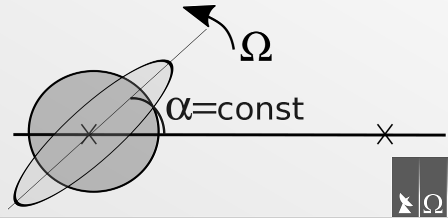
Tidal Timescales

- Equilibrium tides vs Dynamical tides
- Simple Bulge model
- (part of) Hut's more sophisticated model

  (other models Zahn, Tassoul, Eggleton etc.)
- Dissipation mechanisms

Equilibrium Tide

- Without dissipation bulges would line up with M1-M2 axis i.e. instantaneous reaction to the force
- Reality: dissipation exists! (timescale)
- Assume an equilibrium situation with constant angle between the tidal bulges and M1-M2 axis



Equilibrium Tide 2

- Simple bulge model gives $\delta M \sim M_2 \left(\frac{R}{a}\right)^3$
- Hence Torque $\Gamma = -\frac{GM_2^2}{R}\left(\frac{R}{a}\right)^6 \sin\alpha$
- Assume $\alpha \sim \Omega - \omega$
- More accurately $\alpha = \frac{(\Omega-\omega)}{\tau_{\rm diss}}\frac{R^3}{GM}$
- Hence $\Gamma = -\frac{\Omega-\omega}{\tau_{\rm diss}} q^2 M R^2 \left(\frac{R}{a}\right)^6$

Tidal Timescales

- e.g. from Zahn 197*x*

$$\tau_{\rm sync}^{-1} = \frac{-\Gamma}{I\Omega} = \frac{1}{\tau_{\rm diss}}\frac{\Omega-\omega}{\Omega} q^2 \frac{MR^2}{I}\left(\frac{R}{a}\right)^6$$

$$\tau_{\rm circ}^{-1} = \frac{\dot{e}}{e} = \frac{1}{\tau_{\rm diss}}\left(9 - \frac{11}{2}\frac{\Omega}{\omega}\right) q(1+q)\left(\frac{R}{a}\right)^8$$

- As $R \to a$ tides become important
- Valid only for small *e*
- *Sync* is faster than *circ*
- e.g. Moon *e=0.05*

General formulae

- Hut 1981 (*Astronomy and Astrophysics* 99, 126)

$$\frac{da}{dt} = -6\frac{k}{T}q(1+q)\left(\frac{R}{a}\right)^8 \frac{a}{(1-e^2)^{15/2}}\left\{f_1\left(e^2\right) - \left(1-e^2\right)^{3/2} f_2\left(e^2\right)\frac{\Omega}{\omega}\right\}$$

$$\frac{de}{dt} = -27\frac{k}{T}q(1+q)\left(\frac{R}{a}\right)^8 \frac{e}{(1-e^2)^{13/2}}\left\{f_3\left(e^2\right) - \frac{11}{18}\left(1-e^2\right)^{3/2} f_4\left(e^2\right)\right\}$$

$$\frac{d\Omega}{dt} = 3\frac{k}{T}\frac{q^2MR^2}{I}\left(\frac{R}{a}\right)^6 \frac{\omega}{(1-e^2)^6}\left\{f_2\left(e^2\right) - \left(1-e^2\right)^{3/2} f_5\left(e^2\right)\frac{\Omega}{\omega}\right\}$$

f_1 to f_5 are polynomials in e^2

$T =$ tidal timescale

More sophisticated approach?

- e.g. Zahn, also Eggleton (1998 and his book)
- Expand stellar potential in Legendre polynomials to some order (usually 4)
- Calculate deformed structure
- Hence energy dissipation rate
- Very complicated. Too difficult for me!
- But you can try Eggleton, Kiseleva & Hut 1998 (Astrophysical Journal 499, 853)

Dissipation

- Energy is dissipated – how?
- Microscopic diffusion inefficient: $\tau \sim \frac{R^2}{\nu}$

  $\nu = 10 - 1000\,\mathrm{cm^3\,s^{-1}}$
- Turbulent viscosity is faster
- Convection is very efficient $\tau_{\rm conv} \simeq \left(\frac{MR^2}{L}\right)^{\frac{1}{3}}$

$$\tau_{\rm diss} = \frac{\tau_{\rm conv}}{6\lambda} \qquad \lambda \sim 0.02\alpha_{\rm MLT}^{\frac{4}{3}}$$

(Zahn 1989) + corrections if $\tau_{\rm diss} \sim P$

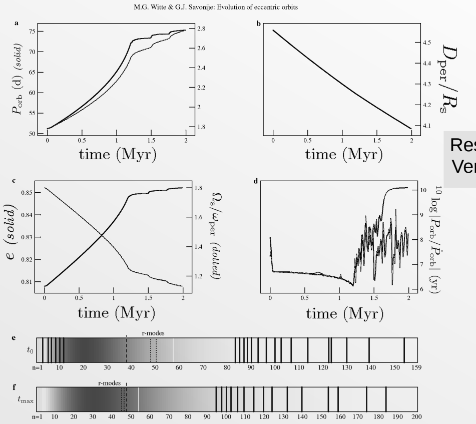
Dynamical Tides

- Pulsation modes excited by companion star
- Dissipative waves e.g.

Mode type	Restoring force
Acoustic	Gas pressure
Gravity	Buoyancy
Inertial	Coriolis

- Adiabatic standing waves in radiative envelope
- Couple core to tidal potential
- Dissipation at the surface: surface torque

Witte & Savonije 1999



Resonance locking:
Very complicated :(

e.g. Goldreich & Nicholson 1989

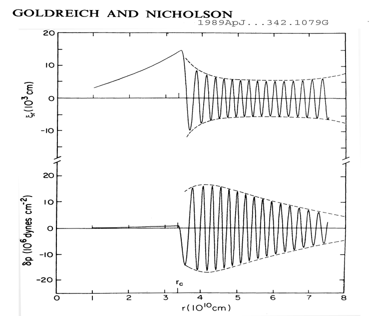
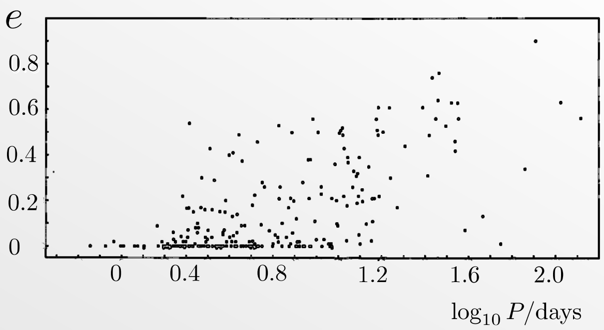


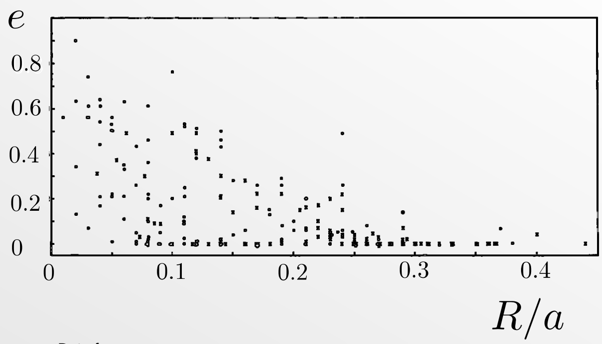
FIG. 1.—Numerical solution of the linearized equations of motion for the dynamical tide in a nonrotating 5 $M_\odot$ main-sequence star. The core-envelope boundary is denoted by r_c. The stellar radius is 1.88×10^{11} cm; the tidal frequency $\sigma = 2 \times 10^{-5}\,\mathrm{s^{-1}}$. Only the homogeneous solution is shown; that is, $U = 0$. The radial displacement and pressure perturbations are denoted by ξ_r and δp. The dashed curves indicate the WKBJ amplitudes obtained from equation (21) and are fitted to the numerical solution at large r (after Fig. 7 of Nicholson 1978).

e vs *P* (unevolved binaries)



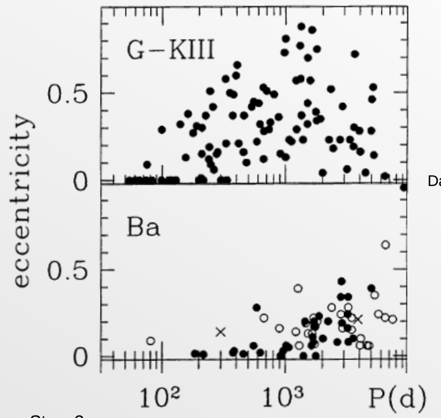
Data from
Giuricin, G.; Mardirossian, F.; Mezzetti
Astronomy and Astrophysics 134, 365

e vs *R/a* (unevolved binaries)



Data from
Giuricin, G.; Mardirossian, F.; Mezzetti
Astronomy and Astrophysics 134, 365

E vs log*P* **(evolved binaries)**



Data from Alain Jorissen

Tides Overview

- Tides synchronise, then circularise
- Rate
- Close binaries should be sync. and circular
- Assuming $\Omega = \omega \text{ and } e = 0$
  we continue our analysis by moving to close, circular binaries and interaction by exchange of *angular momentum and mass*