Binary Stars - astro8501 - 6944

Problem Sheet 4

1. Making reasonable assumptions, estimate the distance between the suns of the planet Tattooine. How close are they to filling their Roche lobes? Is Tattooine in a stable orbit?

2. The Roche potential in a binary-star system is given by

$$\Phi(\mathbf{x}) \quad = \quad \frac{-GM_1}{x_1} - \frac{GM_2}{x_2} - \frac{1}{2}\Omega^2 s^2$$

where x_1 and x_2 are the distances to the two stars, which are located on the x axis, s is the distance to rotation (z) axis. Sketch Φ along the x, y and z axes near the binary system. Locate all turning points, including L_1, L_2 and L_3 on your sketches. Where are L_4 and L_5? Comment on the (3D!) stability of each turning point. In the case $M_1 = M_2 = 1\,\mathrm{M}_\odot$, what is $\Phi(L_1)$?

3. Derive a form of the Roche potential as a function of $\mathbf{x}$, q and a (as shown in the lectures). In the case $q = 1$ derive a quintic equation describing the location $\mathbf{x}_1 = (x_1, 0, 0)$ of the L_1 point. Can you solve it analytically and if not, how would you go about solving the equation? From your sketches, or by calculating derivatives explicitly, is the L_1 point completely stable? What are the implications for spacecraft at the L_1 point?

4. Show that the ratio of the Roche-lobe radii of two stars in a binary system is approximately $R_{L1}/R_{L2} \approx q^{0.46}$ and give a range of q over which your approximation is valid ($q = M_1/M_2$, hint use Eggleton's second formula). Repeat for the other two formulae given for R_L/a in the lecture.

5. Show that in a semi-detached binary the orbital period is related to the average density of the Roche-lobe filling ($R = R_L$) component by

$$P \quad \propto \quad \frac{1}{\sqrt{\bar{\rho}/\bar{\rho}_\odot}} \cdot \frac{1}{(1+q)^{0.2}}$$

and derive the constant of proportionality. Hence comment on the relation between the compactness of the Roche-overflowing star and the period of the system.

6. Use the *binary_c online* tool (`http://www.astro.uni-bonn.de/~izzard/cgi-bin/binary3.cgi`) to evolve a few binaries and complete the following table (note that M_1, M_2, a and P are the initial values of component masses, orbital separation and orbital period, time is measured relative to the

birth of the binary star)

$M_1/\mathrm{M}_\odot$	$M_2/\mathrm{M}_\odot$	$a/\mathrm{R}_\odot$	P/d	Time of RLOF	Primary stellar type at RLOF	Mass transferred/$\mathrm{M}_\odot$	Stellar types after RLOF
2	1	100					
2	1	10					
2	1	3					
20	15	100					
20	15	35					
20	15	23					

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