Binary Stars - astro8501 - 6944

Problem Sheet 5

1. The rate of mass flowing through the L_1 point is given by

$$\dot{M} \approx (\rho v)_{L_1} S$$

where v is the velocity, ρ the density and S the surface area. Show that near the L_1 point, if $\Phi(x,y)$ is the Roche potential,

$$\begin{aligned}\Delta\Phi &= \Phi(x,y) - \Phi(x,0) \\ &\approx \frac{1}{2}\Omega^2 y^2\end{aligned}$$

where Ω is the orbital angular velocity around the z-axis (i.e. the geometry is as in the lecture).
Consider a point far from L_1 on the donor star and explain why

$$\Delta\Phi = \frac{GM_\mathrm{d}}{R_\mathrm{d}}\frac{\Delta R}{R_L}$$

where $\Delta R = R_\mathrm{d} - R_L$, R_d is the donor star radius and R_L is the Roche radius.
Find an expression for S as a function of the previously introduced variables.
By equating $v = c_s$, the speed of sound, show that for a polytropic equation of state

$$\rho v \propto c_s^{\frac{\gamma+1}{\gamma-1}},$$

where γ is the adibatic index. Hence derive a mass-transfer rate for convective stars ($\gamma = 5/3$).

2. An approximate formula for the sound speed is $c_s = 15\,T_4^{-\frac{1}{2}}\,\mathrm{km\,s^{-1}}$ where T_4 is the temperature in units of $10^4\,\mathrm{K}$. Estimate c_s at the surface of an M and O type star. Estimate the orbital velocity v_orb of a close binary star as a function of M_1, M_2 and P. For solar-like components, at what orbital period does $c_s = v_\mathrm{orb}$? Repeat the calculation for an M type star orbiting a white dwarf and a pair of O-type stars. (This will be useful for later in the course!)

3. Let a star have (initial) mass M, radius R, angular velocity ω_0 and moment of inertia $I = kM_0R^2$ where R and k are considered to be constants. By considering the forces on a test particle at the stellar equator, what is the fastest (i.e. critical) angular velocity ω_crit at which the star can rotate? Assume material is accreted from a Keplerian disk at the equator. Derive an expression for the amount of angular momentum accreted ΔJ when a small amount of mass ΔM is accreted correct to $\mathcal{O}\left((\Delta M)^2\right)$ (assuming the stellar structure does not change appreciably). Hence derive an expression for the maximum ΔM which can be accreted by the star assuming its rotational velocity does not exceed the breakup velocity and that it starts from rest i.e. $\omega_0 = 0$. Comment on what the result would be if $\omega_0 > 0$ and on the validity of the assumptions $R =$ constant and $k =$ constant.

4. Show that the angular momentum in a Keplerian disc of constant density is given by

$$J_\mathrm{disc} = \frac{4}{5}\sqrt{GMR_d}M_\mathrm{d}\,.$$

Approximately what is the ratio of the angular momentum stored in an accretion disc to that in the orbit of a binary-star system when one of the stars is transferring mass through the L_1 point and the stream does not directly impact the companion?

Questions, problems, errors? Contact Rob Izzard by email: izzard@astro.uni-bonn.de