# *Binary Stars - astro8501 - 6944*

**Problem Sheet 7**

1. *Bondi Accretion*: spherically symmetric inflow of gas onto a stationary accretor.
Consider a mass $M$ embedded in a gas which is at rest at infinity with density, pressure and sound speed there of $\rho_\infty$, $P_\infty$ and $c_\infty = \sqrt{dP/d\rho|_\infty}$ respectively. Assuming a polytropic equation of state $P = A\rho^\gamma$, the continuity and momentum (Euler) equations governing the flow are

$$\nabla \cdot \rho \boldsymbol{u} = 0 \quad (1)$$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2}\,, \quad (2)$$

where $v$ is the inward radial velocity and $\boldsymbol{u}$ the 3D velocity vector. Rewrite these in terms of spherical polar co-ordinates and $d\rho/dr$ instead of $dP/dr$.

- By integrating Eq. 1, derive the mass accretion rate $\dot{M}_{\mathrm{acc}}$ and its dependence on radial co-ordinate $r$.
- Show that $\rho = \rho_\infty \left(c_s/c_\infty\right)^\omega$ for any point in the flow and find $\omega$.
- Integrate the alternative form of the Euler equation (Eq. 2) to derive the Bernoulli equation (also derive any constants of integration in terms of variables given above).
- Consider solutions such that $v$ monotonically increases from 0 at infinity to free-fall speeds ($v \to \sqrt{2GM/r}$) near the origin. Show that Eq. 1 can be rewritten as

$$\frac{\rho'}{\rho} + \frac{v'}{v} + \frac{2}{r} = 0$$

and Eq. 2 can be rewritten as

$$vv' + c_s^2\frac{\rho'}{\rho} + \frac{GM}{r^2} = 0\,,$$

where $'$ means $d/dr$. Hence find expressions $v'$ and $\rho'$ and show that the flow, if it is to remain monotonically increasing in velocity, it must pass through a *critical point* and identify $r$ and $v$ at the critical point.
- Evaluate Bernouilli's equation at the critical point to find the location and sound speed at the critical point in terms of the conditions at infinity.
- Evaluate $\dot{M}$ at the critical point and hence confirm Bondi's result $\dot{M} \propto \rho_\infty M^2 c_\infty^{-3}$.

2. Two stars in a binary system are labelled $M_w$ and $M_a$. These represent a star losing a fast wind of velocity $v_w$ ($\gg v_{\mathrm{orb}}$, with radius $R_w$) and a star accreting that wind (with radius $R_a$) respectively. The mass accretion rate is given by, under the assumption of a fast wind,

$$\dot{M}_a = -G^2\frac{\dot{M}_w M_a^2}{v_w^4}\frac{1}{a^2\sqrt{1-e^2}}\,.$$

- Estimate $v_w$ hence and substitute into the expression for $\dot{M}_a$.
- Assume the mass-losing star is a thermally-pulsing AGB star of mass $3\,M_\odot$, estimate the mass accreted, as a function of the separation $a$, by a solar-like companion by the time the mass-losing star becomes a white dwarf (assuming a circular orbit).
- A typical barium-star system has $P = 2000\,$days, estimate the amount of mass accreted by the barium star (you will need to make some simplifying – but reasonable – assumptions).

3. If both stars in a binary have fast winds with mass loss rates $M_1$ and $M_2$, velocity $v_1$ and $v_2$, relative to the circular orbital motion, estimate the location of the shock where the winds meet. What is the condition that there is accretion of material from star 1 onto the surface of star 2? Estimate the luminosity of the shocked material – when will this be significant?

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