Binary Stars - astro8501 - 6944

Problem Sheet 9

1. Consider the equations of motion for a fluid element of unit mass in a thin, circular accretion disc with velocity $\Omega(R)$ where R is the distance from the centre of the disc (i.e. the rotation axis). Write down the centripetal force required to keep the particle in orbit and hence write the equation of motion assuming a central mass M.

   Now consider the motion of the fluid element in a co-ordinate frame rotating at $\Omega(R_0) = \Omega_0$ with the origin at the location of the mass element. We will consider motion as a function of distance x from R_0 (pointing radially outwards away from the rotation axis) where $x \ll R_0$. By utilising the Taylor series for $\Omega^2(R_0 + x)$ around R_0, or otherwise, write down the sum of the centripetal and centrifugal accelerations to $\mathcal{O}(x)$ and relate these to the spatial derivative of Ω^2, $d\left(\Omega^2\right)/dr$.

   The x axis is defined to point radially outwards and y along the azimuth (i.e. along the unperturbed orbit). Derive the equations of motion assuming an arbitrary force (f_x, f_y) acts in the x and y directions on the fluid element, e.g.

$$\ddot{x} = f_{\mathrm{centrifugal}} + f_{\mathrm{centripetal}} + f_{\mathrm{Coriolis}} + f_x\,.$$

   Investigate the stability of the case $f_x = f_y = 0$ (e.g. pressure-free "ballistic" motion) by considering solutions of the form $x = x_0 e^{i\omega t}$ and $y = y_0 e^{i\omega t}$, hence derive the dispersion relation

$$\begin{aligned}\omega^2 &= R\frac{d}{dR}\Omega^2 + 4\Omega_0^2 \\ &= \kappa^2\end{aligned}$$

   and comment on stability of the oscillations and their relation to the specific angular momentum of the disc.

   Consider the case of an external force $f_x = -Kx$ and $f_y = -Ky$ where K is a "spring constant" and derive the dispersion relation

$$\omega^4 - \omega^2\left(2K + \kappa^2\right) + K\left(R\frac{d}{dR}\Omega^2 + K\right) = 0\,.$$

   What is $d(\Omega^2)/dR$ for a Keplerian disc? Is it stable to the magnetorotational instability?

Questions, problems, errors? Contact Rob Izzard by email: izzard@astro.uni-bonn.de