Stars and Stellar Evolution
Computer Labs
Winter Term 2014-15
with
Window To The Stars

http://www.astro.uni-bonn.de/~izzard/stellar_computing.html
http://www.astro.uni-bonn.de/~nlanger/siu_web/teach_sse.html

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1 Introduction

The standard computational tool of anyone interested in understanding stars is a stellar evolution code — a piece of software that can construct a model for the interior of a star, and then evolve it over time. Evolution codes allow us to check and refine the various physical theories that together compose stellar astrophysics (e.g., atomic physics, nuclear physics, fluid dynamics, thermodynamics); they provide laboratories for performing experiments on stars (e.g. discovering what factors contribute to the formation of red giants); and, they shed light on stages of stellar evolution that may be too fleeting to observe directly in the Universe.

Stellar evolution codes trace their ancestry back half a century, to a seminal paper by Henyey, Forbes & Gould (1964). With the advent of electronic computers, these authors devised a way to solve the partial differential equations governing stellar structure and evolution. In these equations, spatial gradients are replaced by finite-difference approximations represented on a discrete grid of points, extending from the center of the model to the surface. This leads to a large set of simultaneous equations for the state variables (pressure, density, etc.) at each grid point, which can be solved using matrix methods.

Many evolution codes have been written based on the Henyey method, and various improvements to the method have been introduced over time. Among the most well-known historical codes are those by Eggleton, Kippenhahn and Paczynski — quite a few modern codes are essentially heavily modified versions of these. The Eggleton (1971) code was particularly innovative, in that it introduced an algorithm for automatic redistribution of grid points. This allows stars to be evolved up the red giant branch with only a few hundred points in total, which makes the code very fast.

One descendant of the original Eggleton code is the TWIN code, which allows simulation of binary evolution as well as single-star evolution. While the physics in TWIN is quite up to date, it is very user-unfriendly. To address this issue, Robert Izzard and Evert Glebbeek have developed Window To The Stars (WTTS), a graphical interface to TWIN, which will be used for this course. All relevant information regarding this course and the WTTS software is available for free online at: http://www.astro.uni-bonn.de/~izzard/stellar_computing.html

2 Background

Stellar evolution is based upon the solution of the equations of stellar structure, together with equations governing the mixing and nuclear burning of chemical elements. For a spherically symmetric star that is not rotating, these equations are:

- the equation of hydrostatic equilibrium:

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4},$$ (1)

  where P, r and m are pressure, radius and the mass contained within a spherical shell of radius r, respectively,

- the equation of mass conservation:

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho},$$ (2)

  where ρ is the density: $dm = 4\pi r^2 \rho dr$,

- the equation of energy generation:

$$\frac{dL}{dm} = \epsilon,$$ (3)

  where L is luminosity and ϵ is the energy generation rate including nuclear energy generation, energy from gravitational sources and energy losses from neutrino emission, and

- the equation of energy transport, which may be expressed as:

$$\frac{d\ln T}{dm} = -\nabla \frac{d\ln P}{dm}$$ (4)

  where the form of ∇ depends on whether the region of the star is radiative or convective (refer Chapter 4 from the lecture notes).

Equations 1-4 determine the (static) structure of the star and hence are referred to as the "structure equations". In order to model the star's evolution we also need to track the composition of the star which is altered by nuclear reactions and also by material being mixed throughout the star by convection. If mixing is assumed to be modeled as a diffusive process, the change in mass fraction X_i of the element i is governed by the equation (same as Eq. 6.5 in the lecture notes):

$$\frac{d}{dm}\left(\sigma \frac{dX_i}{dm}\right) = \frac{dX_i}{dt} + R_i - S_i$$ (5)

where σ is the diffusion coefficient, R_i is the rate at which the species i is being burnt by nuclear reactions and S_i is the rate at which it is being produced by nuclear reactions.

The remaining physical inputs include the equation of state for the material, data covering reaction rates, neutrino losses and opacity of the material.

Now, in order to understand the method for (simultaneously) solving the above equations, it is essential to be familiar with the following terminology:

- A **time-step** is defined as the act of moving from a model at time t to one at time $t + \triangle t$.

- To produce a model at a new time-step using the relaxation method (Heyney method: refer Kippenhahn & Weigert Chapter 11), it is necessary to make **iterations** on the solution.

- When the changes to the current solution are sufficiently small the model is said to have **converged**.

The "fully-simultaneous" method involves solving all the equations (1-5) together at each individual iteration of each time-step, thus giving the internal structure i.e. $\rho(m, t)$, $P(m, t)$, $T(m, t)$, $r(m, t)$, $X_i(m, t)$ at every time-step. Given an initial model e.g. $\rho(m, t)$, it is possible then to compute the next model (given a fixed timestep $\triangle t$) i.e. calculate $\rho(m, t + \triangle t)$. This is essentially the task for any stellar evolutionary code, as will be illustrated with the following exercises using WTTS.

3   Stellar model sequences

Stars are modelled as one-dimensional spheres. Even in a star rotating so fast it is about to break apart, this approximation is not too bad (to within 50% in the radius). The star is divided into many *shells* labelled by their mass co-ordinate m (where $0 \leqslant m \leqslant M$, where M is the total mass of the star).

- A stellar *model* is a description of the state of a star (pressure, density, temperature, radius etc.) for *one time only,* usually labelled t.

- A stellar *model sequence* is for many different consecutive times, t, $t + \Delta t$, ...

WTTS uses a *starting model* – usually of a "zero-age main sequence" (ZAMS) star which has just started to burn hydrogen – to construct a *model sequence* at later times. Given that at time t we have a model of $\rho(m, t)$, $P(m, t)$, $T(m, t)$, $r(m, t)$, $l(m, t)$, $X_i(m, t)$ etc. inside the star, we want to calculate $\rho(m, t + \Delta t)$, $P(m, t + \Delta t)$, $T(m, t + \Delta t)$, $r(m, t + \Delta t)$, $l(m, t + \Delta t)$, $X_i(m, t + \Delta t)$. At each shell we have a *stellar structure equation*, so if there are N shells, there are (at least!) $4N + N_{\text{isotopes}}$ equations (one for each species) to be solved. Given that $N \gtrsim 200$ for a realistic and accurate stellar model, this implies at least 2000 coupled equations must be solved at each timestep. Note that there are 5 equations mentioned above but 6 variables: the final equation to be solved is the *equation of state* which gives $P(\rho, T)$, or equivalently $T(\rho, P)$ or $\rho(P, T)$. At the surface and the core, *boundary conditions* replace the normal equations: there are two boundary conditions at the core, and two at the surface, which replace the four stellar structure equations.

Model Relaxation

The *relaxation method* is not about going to sleep! Read all about such methods at
https://en.wikipedia.org/wiki/Relaxation_method.

   The method employed by the TWIN code is the Henyey scheme (see Kippenhahn & Weigert Chapter 11 or the talk at http://www.astro.uni-bonn.de/~izzard/technical.html). The equations are *discretized* (see https://en.wikipedia.org/wiki/Discretization) and *relaxed* (or *solved*) to a stated accuracy, at which point they are said to be *converged*. The process is repeated and the stellar *model sequence* stops either when a maximum time is reached (or some other stop condition is true), or when convergence is impossible.

4   Exercises

Perform the following steps (in the order as below) in order to carry out the exercises (1-6):

1. Login using your **uni-bonn account** (NOTE: use **XFCE**):
   For example, if you are *fritz@uni-bonn.de* then your login name is *fritz*.

   You will need to work from a *terminal*, so open one using the icon at the bottom of the screen or the system menu. You may have to change the colour settings in the terminal in order to see text properly.

2. Create a parent directory in your home directory to run and save your models/results using WTTS. This could be SSE_WTTS for instance, in which case you would run:

   mkdir SSE_WTTS

   Always work from within this parent directory.

3. Each task you perform should be from **within a subdirectory of SSE_WTTS with a unique name**, e.g. if you want to evolve a 1 M$_\odot$ star, and call the directory *1msun*, run the following:

   cd SSE_WTTS
   mkdir 1msun

   Launch WTTS from within the appropriate subdirectory:

   cd 1msun
   /vol/software/software/astro/wtts/wtts

4. To evolve a new (mass) model sequence, start WTTS (following Step 3). Set appropriate inputs (as required for the task/exercise) **before you hit Evolve.** Note that when you hit Evolve, any model sequence in the current working directory will be erased and the overwritten!

5. To reload (NOT TO EVOLVE) an existing model sequence (e.g. from a previous class) simply launch WTTS (Step 3) from the appropriate subdirectory and use the different tabs (e.g. HRD, Structure,..) to plot/calculate results required for each exercise. Do not hit the evolve button!

6. For the following exercises (to be done in order) *save all your plots* by right-clicking and using "Save As Postscript" (or, if using the PNG plotting option, "Save As").

This is a *laboratory course*. **You are expected to keep notes and save your work.** You should keep your model sequences. You should keep *backups* in case of computer failure (trust nobody to do this for you!).

5   Final Report

You are required to write up the answers to the course questions as a written report in which you include evidence that you have done the work and achieved the aims of the exercise. This is not optional. The deadline for the report will be posted on the course website[1]. The report is to be *yours alone* – do not copy others and, although you are free to help each other in class, you will be assessed separately and individually. Any work found to be copied from somebody else will lose you (and possibly them) marks and you may have to repeat the class. The grade from this course contributes **30%** of your stars and stellar evolution course grade, so take it seriously. Please, to avoid excessive workload at the end of the semester, *when you will be very busy with exams*, write up the work *as you go along*. Take good notes, organise yourself from day one like a professional scientist. If you are smart, you'll use LaTeX, if you're even smarter you'll use LyX (www.lyx.org). **Please also make sure your report follows *exactly* the layout of the exercises**, i.e. exercise 1, then 2, then 3... thanks!

   Finally, remember, we're here to help! Just ask when you need something. Help is provided only during the lab time, so in order to get the most out of the course, you have to turn up. The "early" mornings suck, but for us too!

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[1]http://www.astro.uni-bonn.de/~izzard/stellar_computing.html

Exercise 1.

1. Evolve a $1\,M_\odot$ ($Z = 0.02$) model through the main-sequence phase and toward the giant branch. You will have to "Terminate" the evolution at some point, but you should be able to reach luminosities at least $10\,L_\odot$ and perhaps more in the time allowed.

   a) What are the effective temperature and luminosity of the *starting* model?

   b) What are the maximum effective temperature and luminosity it eventually reaches?

2. Locate the point in the *HRD* (go to the *HRD* tab) corresponding to the same luminosity as our Sun has now.

   a) What is the approximate age? (Be accurate to $\pm 0.1\,\mathrm{Gyr}$)

   b) What is the effective temperature?

   c) Is this a good model for the Sun?

3. Go to the *Structure* tab.

   a) Which model has the Sun's luminosity? What is its age?

   b) How hot is the centre of the star at age zero?

   c) What temperature does the star reach in the final model?

   d) What is the central temperature in the model most like our Sun?

   e) How long does it take for the star to exhaust its central H supply?

   f) How and why does this affect the central temperature?

4. Using the Structure tab, plot the radius of your star as a function of time.

   a) When does the star pass through the current solar radius?

   b) Based on (a), how would you rate this as an appropriate model of the Sun?

   c) Can you think of any effect that could improve this? OPTIONAL: try adjusting the convection parameter CALP in the mixing tab (under options).

5. Select Central Abundances of C, N, and O and plot them (you might want to use logY axis) as a function of time.

   a) What are the initial abundances of C, N and O? What is their sum i.e. $C + N + O$?

   b) Why does the carbon abundance drop quickly at early times (you might want to chose log x axis to see this) ?

   c) What does the carbon turn into and which burning cycle is involved? What is the main product of this burning cycle?

   d) What happens to the oxygen abundance after 6Gyr? Why does it not happen earlier? Hint: recall reaction cross-sections and the Gamow peak

   e) What does the oxygen turn into and which burning cycle is involved? What is the main product of this burning cycle?

   f) What is the sum $C + N + O =$ at $t = 0, 5, 10\,\mathrm{Gyr}$? Why is this sum (almost) a constant? Why is it not (quite) constant?

Exercise 2.

1. Evolve a high mass model, $M = 15\,M_\odot$. Plot the time evolution of the central temperature and central abundances of the different products of nuclear burning (choose appropriate scales for your plots).

2. Plot log central density vs log central temperature from the RhoT tab.

   a) Identify the onset of various burning stages in the course of evolution of such a star (e.g. by labelling the central helium abundance).

   b) What are the central temperatures corresponding to these nuclear burning episodes?

   c) Explain the behaviour of neon at the centre of the star.

3. Compare this with the $1\,M_\odot$ model.

   a) What are the timescales (in Myr or Gyr) for the different nuclear burning phases in both stars?

   b) Why are the timescales so different between a $1\,M_\odot$ and $15\,M_\odot$ star?

Exercise 3.

Using **the Kippenhahn tab** for the $1\,M_\odot$ and $15\,M_\odot(Z = 0.02)$ models , answer the following (comparing the plots for the two masses) :

1. Plot *Age* on x-axis, *Mass* on y-axis and *H* (hydrogen mass fraction) as the z-axis. Explain what you see.

2. Plot *Age*, *M* and *E_nuc* (the last as log 10). This shows the nuclear burning regions.

   a) What happens to the burning region when the core runs out of hydrogen?

   b) What happens to the magnitude of the burning? (Hint: try setting the y range to zoom in on the central region.)

3. Change *Age* to *Model Number*. **Replot**.

   a) Why is it easier to see the details of the transition from core to shell burning when plotting against Model Number?

   b) Why is the Model Number not simply linearly proportional to time?

4. Change back to *Age* for the x-axis and try *L* on the z-axis (linear).

   a) Can you explain what you see?

   b) Compare the maximum luminosities with the values in the HRD.

Exercise 4.

Move back to the $1\,M_\odot$ star and make sure you have evolved this star up the giant branch until the evolution automatically finishes. Using the **Internals tab**, answer the following questions by plotting every few models (e.g. every 100 models). Remember not to crowd the plots with too many lines.

1. Plot log(density) vs mass for various models.

   a) There appears to be a point of inflexion near $m = 0.2\,M_\odot$ in some later models. What is happening there?

   b) Plot $\nabla_{rad} - \nabla_{ad}$ vs mass (with appropriate axis ranges and without crowding the plot with too many lines) to determine where the convective regions are. How do these vary with time?

   c) The core becomes *approximately* isothermal after central hydrogen is exhausted. What physical reasons(s) give rise to an *isothermal* core? How much luminosity is generated in this core? By what?

2. Where and when are neutrino losses important? (look at ϵ_ν)

3. Look at the C, N and O abundances. What is happening near the end of the giant branch evolution? Verify your answers by comparing with previous results.

4. Change the abscissa from M to *opacity* (you might want to use log axes).

   a) What is the relation between $\nabla_{rad} - \nabla_{ad}$ and *opacity*?

   b) Is this due to changes in ∇_{rad} or ∇_{ad}? Why do these changes occur?

   c) On the main sequence, at what temperature is the opacity the greatest? Where is this in the star (find the M & R coordinates) ?

Exercise 5.

Load a set of zero age main sequence (ZAMS) models to answer the following:

1. Using the **HRD tab**, create the HRD for the ZAMS models labeled by the mass (chose appropriate colors, line width, label spacing).

    a) What color are the low mass and high mass main sequence stars?

    b) What was the surface temperature of the Sun when it was born? .

2. Using the **Structure tab**, estimate expressions for luminosity and radius as a functions of mass in the forms $L \propto M^x$ and $R \propto M^y$. Justify your answers and show your working: simply copying from the lecture notes will not get you any marks.

3. Go to the **Kippenhahn tab**. Chose Mass (log10) as the x-axis, M/Mass (linear) as the y-axis.

    a) Plot *Convection (Log10)* as the z-axis.
    What is the minimum mass for a star to have a convective core on the ZAMS?
    What is the maximum mass for a star to have a surface convective region?

    b) Plot *logT* instead of convection (Hint: reset the z-range to get better contrast). Why are the higher mass stars hotter?

    c) Plot *rho* as z-axis (also reset z-range appropriately). Explain what you observe in this plot.

    d) Plot *E_nuc (Log10)* on the z-axis and comment on what you observe.

    Describe their relative contribution to the nuclear burning rate (12.1) as a function of mass.

Exercise 6. The Virial Theorem

This exercise is a recap of the virial theorem and its importance in stellar evolution. Just to remind it to you the virial theorem states that,

$$U + 2T = 0 \, ,$$

where T is the kinetic energy of the system and U is the potential energy. In a star, the kinetic energy T is related to the internal energy E_{int} through the adiabatic index γ and gravity is the source of potential energy $U = E_{grav}$, so,

$$3(\gamma - 1)E_{int} + E_{grav} = 0 \, ,$$

where the adiabatic index $\gamma = 5/3$ for an ideal gas. We want to investigate stellar models to test the virial theorem and the validity of the ideal gas approximation during various stages in the life of a normal star (like our Sun) or more massive stars (e.g. the models used in the previous exercise).

1. With your solar mass model perform the following tasks:

  1. Use the **HRD** to select an appropriate model **for the main sequence**.

  2. Using the **Internals tab**, save the model as 1Msun_MS.mdl (just by right clicking on the model number you have chosen).

  3. Download and save the **perl script** "virial_theorem.pl" (from the course webpage) in the directory where you performed the previous step . You now have to edit the script to make it work with your .mdl input file.

     a) Using your favourite text editor, e.g. *emacs* or *gedit*, change the filename in the perlscript i.e. find the line **$input_file="1Msun_MS.mdl";**

     b) Check and make sure you understand the expressions that calculate the internal and gravitational energies, E_{grav} and E_{int}, i.e. **$E_grav -=......;** and **$E_int +=.....;** (Note that *U* is given in the model file as the *specific internal energy*.)

  4. Comment on your results obtained for E_{grav}/E_{int} (from the model) with regards to the validity of the ideal gas approximation (i.e. what is γ for this star?).

  5. How does the α factor to compare with the result you obtained in the exercise 2.3(a) from the stars and stellar evolution lectures? [e.g.$\alpha = 1.5$ for the main-sequence (ref. Maeder)]

2. Using the same prescription, select a **mid-main-sequence model** for a high mass star (e.g. from the previous $15\,M_\odot$ model sequence; justify your choice of model) to find a value of γ for a massive star. What does this imply about the stability of such stars and their equation of state?

3. The total energy of the star is $E_{tot} = E_{int} + E_{grav}$. Derive an expression for E_{tot} as a function of γ and either E_{int} or E_{grav}. From this, deduce what happens if the star has $\gamma < 4/3$.