Population Nucleosynthesis

Rob Izzard

Carolune Institute for Quality Astronomy
www.carolune.net/ciqua

Visitor at Saint Mary's University, Halifax

"At CIQuA we strive to study astronomy at the highest level possible, usually around 1500m."

Collaborators

Chris Tout, Maria Lugaro, Richard Stancliffe, Ross Church (IoA, Cambridge)
Amanda Karakas (Saint Mary's, Halifax, Canada)
John Lattanzio (Monash University, Melbourne, Australia)
Onno Pols, Axel Bonacic (Utrecht, The Netherlands)
Lynnette Dray (Leicester, UK)
Carolina Ödman (CIQuA)

Population Nucleosynthesis

- Detailed single stellar evolution models
- Detailed binary stellar evolution models
- Single vs binary parameter space
- (Traditional) Population Synthesis
- Why use Population Synthesis?
- Population Nucleosynthesis
- AGB stars, Hot Bottoms, Binary Processes
- Chemical Yields, Nuclear Reaction Rates

Detailed 1D Single Stellar Evolution

Solutions to equations:

- Hydrostatic equilibrium $\frac{dP}{dm} = \frac{-Gm}{4\pi r^4}$
- Mass conservation $\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$
- Nuclear energy generation $\frac{dL}{dm} = \epsilon$
- (Radiative) Transport of the energy flux $\frac{dT}{dm} = -\frac{3}{4ac}\frac{\kappa}{T^3}\frac{F}{(4\pi r^2)^2}$ (or convection prescription)
- Chemistry $\frac{dX_i}{dt} = X_i X_j \langle \sigma v \rangle_{ij}$

Run time and extra time

- Code runs in $\sim$ minutes-hours
- Basic chemistry: ^{1}H, ^{4}He, ^{12}C, ^{14}N, ^{16}O, ^{20}Ne, ^{56}Fe
- Post-processing nucleosynthesis required for extra isotopes
- Code runs in $\sim$ days

Single Star Uncertainties

- Initial mass M
- Initial abundances Z
- Mass-loss prescription
- Convective mixing prescription (MLT)
- Nuclear reaction rates $\langle\sigma v\rangle$
- Code breakdown (numerical problems)
- Coupling to supernova II/Ib/c models

Detailed 1D Binary Stellar Evolution

There are two types of code:

Detailed 1D Binary Stellar Evolution

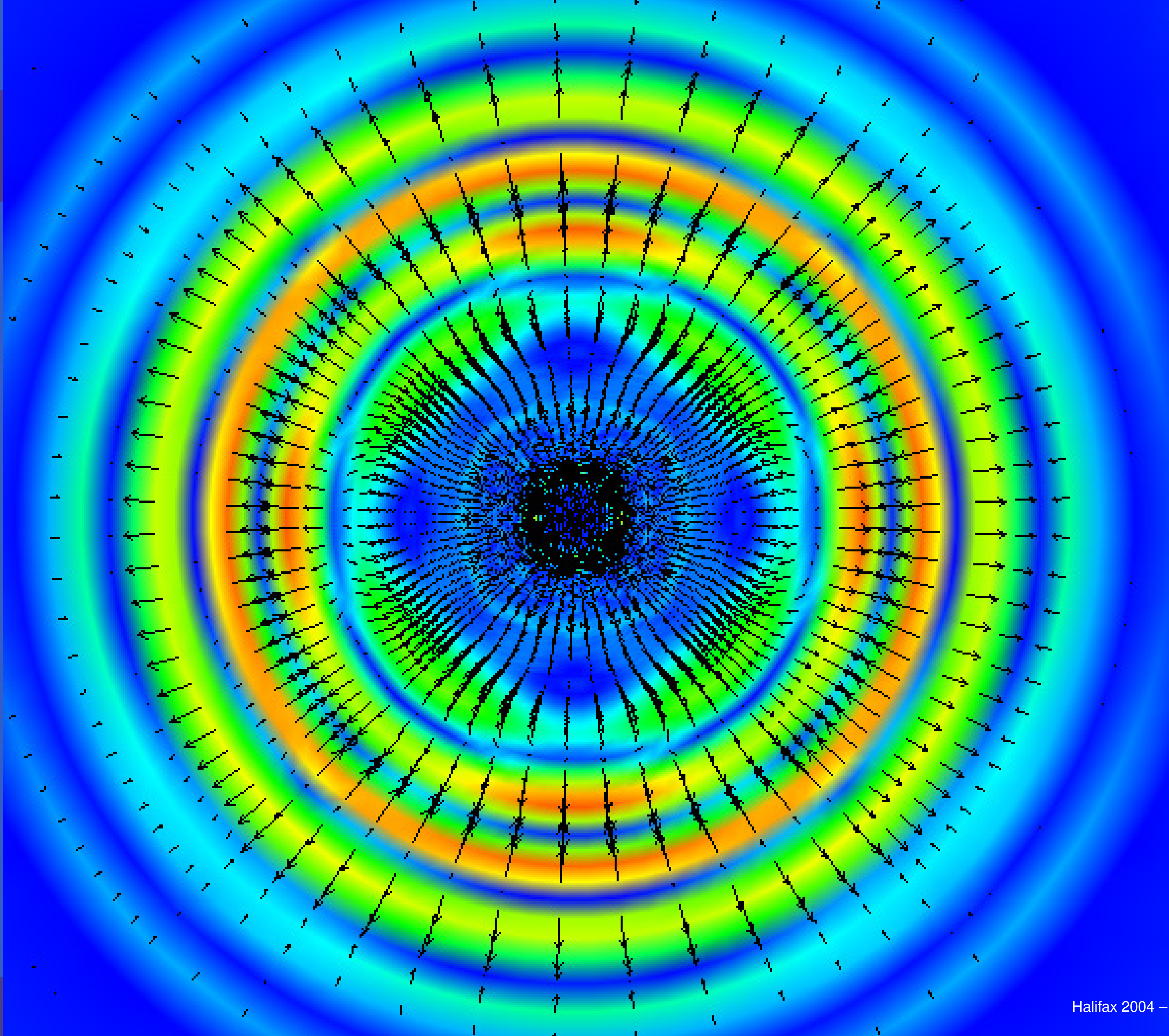
There are two types of code:
Coupled 1D models. e.g. Eggleton's TWIN.

- Assumes stars are approximately spherical
- 1D stellar structure equations
- Perhaps small perturbations
- Simple mass transfer

Detailed 3D Binary Stellar Evolution

Full 3D models, e.g. Djehuty

- 3D explicit (magneto-) hydrodynamic code
- Nuclear bomb simulation code!
- As close to “reality” as we can get
- Requires (US military) supercomputers
- Most of us are not allowed to use it!



Binary Star Uncertainties

Guess. . .

Binary Star Uncertainties

- Initial mass M
- Initial abundances Z
- Mass-loss prescription
- Convective mixing prescription (MLT)
- Nuclear reaction rates $\langle \sigma v \rangle$
- Code breakdown (numerical problems)
- Coupling to supernova II/Ib/c models

Binary Star Processes

- Tidal Interaction
- Roche Lobe Overflow
- Common Envelope Evolution
- Wind collision
- Wind accretion
- Thermohaline Mixing
- Explosions: Type Ia SNe and novae

Run time and extra time

- Code runs in at least twice the time of its single star equivalent
- Basic chemistry: ^{1}H, ^{4}He, ^{12}C, ^{14}N, ^{16}O, ^{20}Ne, ^{56}Fe
- Post-processing nucleosynthesis???
- Djehuty code evolves stars in approximately real time!

Single Star Parameter Space

At its simplest, this is two dimensional

- Mass M (distribution IMF)
- Metallicity Z (solar scaled or LMC/SMC)

Assume $Z = Z_{\odot}$ to reduce this to 1D.
All other physics is fixed.

Binary Star Parameter Space

This is never simple!

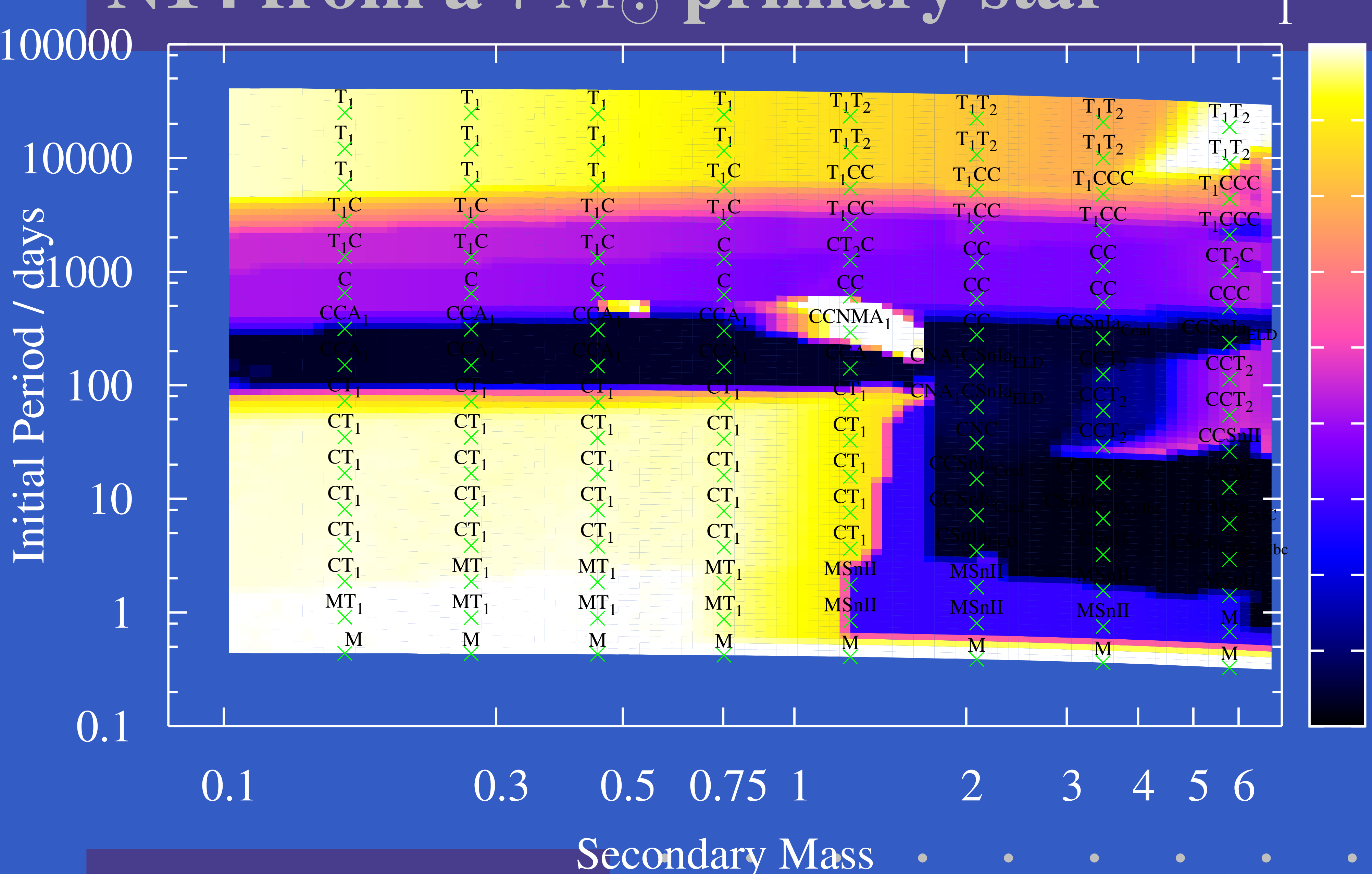
- Mass M_1 (IMF)
- Mass M_2 (or ratio $q = M_2/M_1$, flat-q)
- Metallicity Z (abundances solar scaled?)
- Separation a (or period P related by Kepler's law, flat-$\ln$)
- Eccentricity e

A five-dimensional space!
Assume $e = 0$, $Z = Z_\odot$ to reduce it to 3D.

Chaos in Binaries

- Evolution is chaotic
- Perturb initial conditions slightly $\longrightarrow$
- Leads to very different evolution!
- High resolution grid required to resolve these effects
- Particularly novae and SNe Ia which occur rarely

N14 from a $7\,M_\odot$ primary star



Time is Money (unless you're a studen

- At 1 hour per model (*very* conservative!)

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- Or 10^6 hours...

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- Or 114 years...

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- Or 114 years...
- Or 38 PhDs...

Time is Money (unless you're a studen

- At 1 hour per model (*very* conservative!)
- With 100 grid points per dimension...
- That is 10^6 grid points...
- Or 10^6 hours...
- Or 41667 days...
- Or 114 years...
- Or 38 PhDs...
- That assumes you got the physics right in the first place. Which you didn't!

Population Synthesis

What if we can reduce the code run time by a factor of 10^7? Evolution of a stellar population would take a matter of hours. You could start a model run, go to the pub, and it'll be finished by the time you wake up . . .

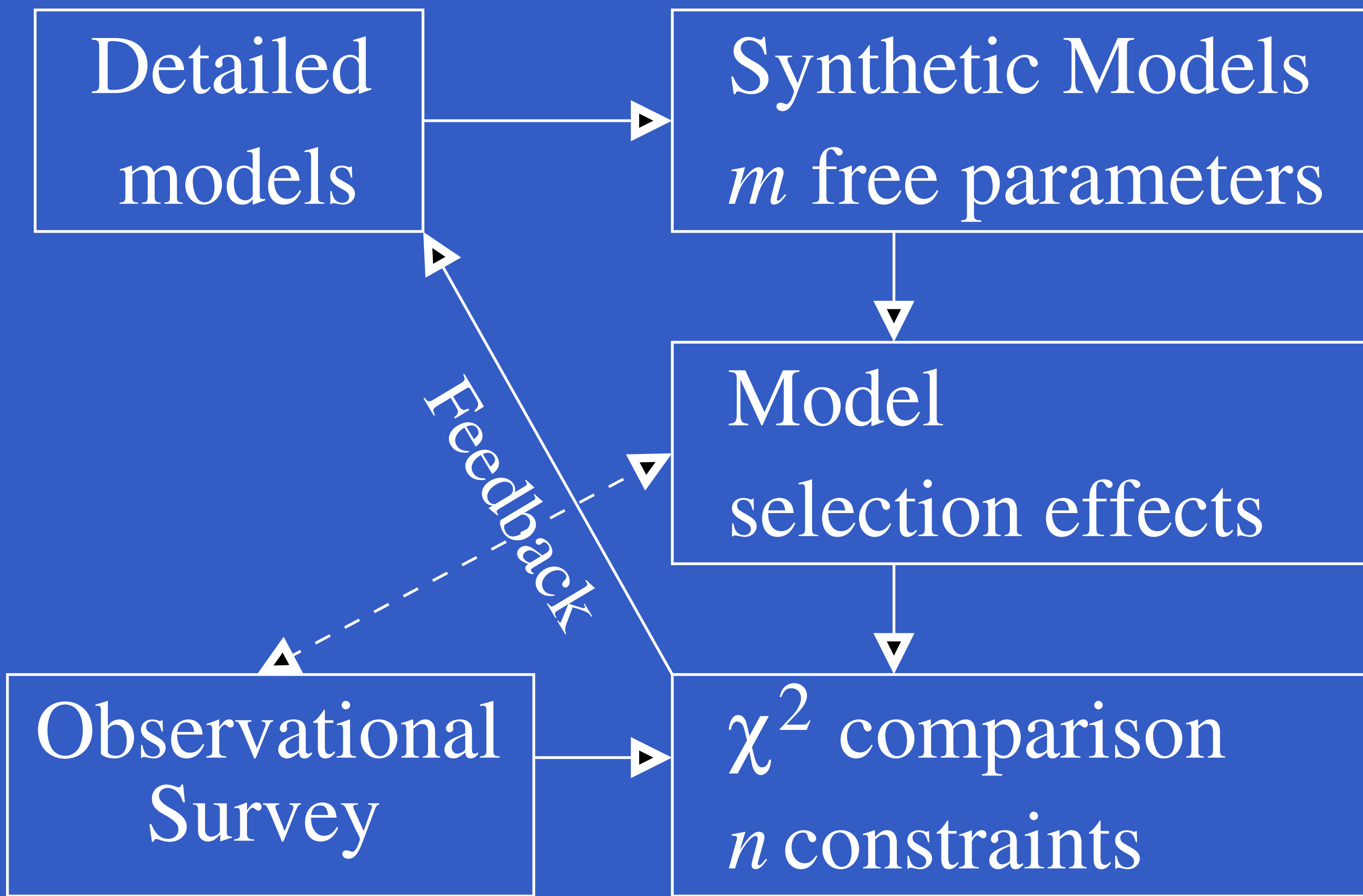
Population Synthesis

- Fit results from detailed models to simple functions
- Fit timescales τ_i, radius R, luminosity L, core mass M_{c} etc. as $f(M, Z, t)$
- Couple to binary star model (tides, RLOF, CE, wind accretion etc.)
- Model of single or binary star, from birth to death, takes $< 0.1\mathrm{s}$
- Millions of stars per day!
- Lose internal stellar structure information.

Why Population Synthesis?

- It is the only way to explore the complete parameter space
- Easily experiment with new physics e.g. change mass-loss prescription, common envelope removal efficiency etc.
- Compare to observations e.g. number ratios of stellar types, supernova rates etc. to determine the value of input parameters
- Feed these results back to detailed modellers
- Tells them what they *should* be getting!

Flow diagram of (my?) life



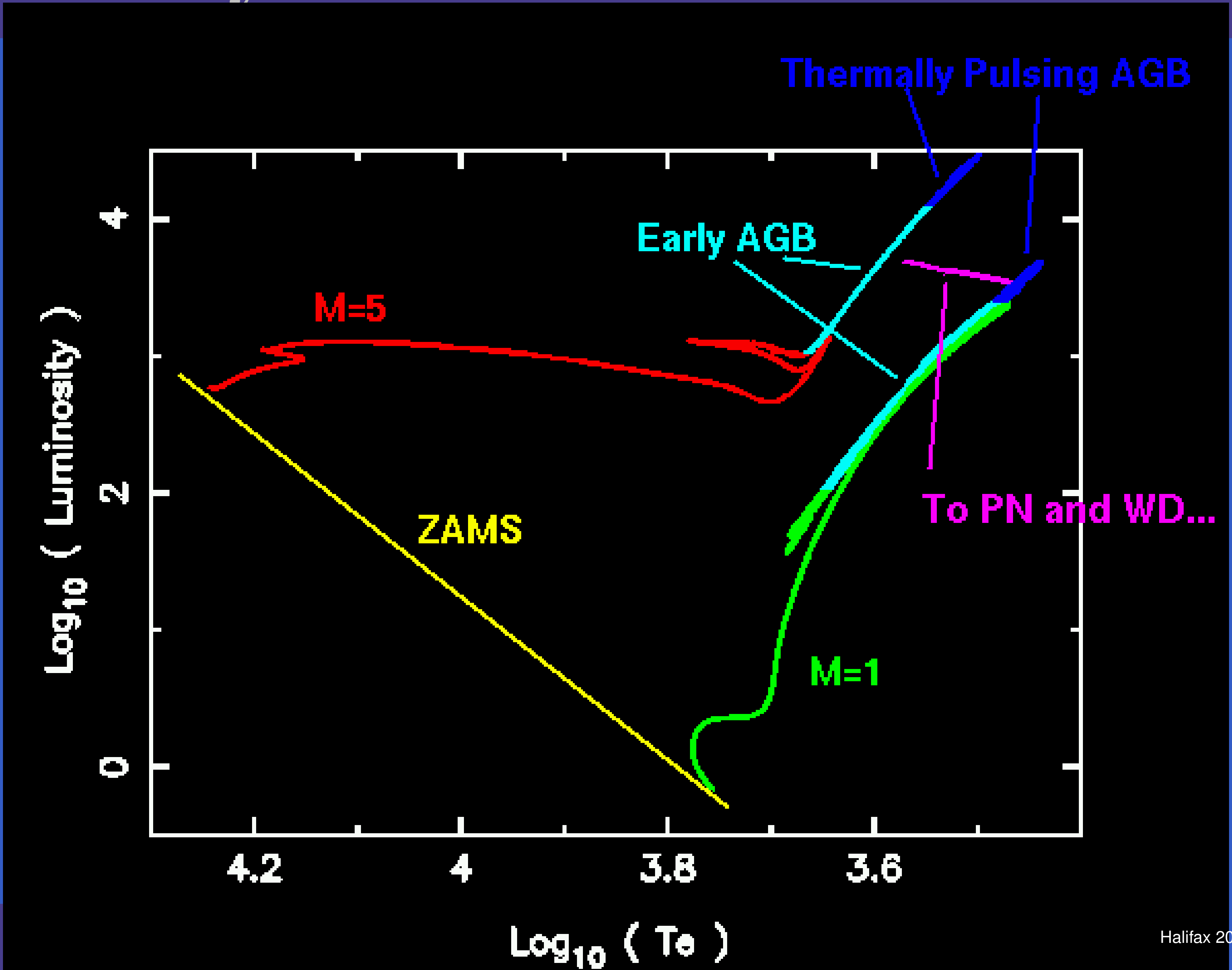
Limitations

- Fit accuracy $\sim 5\%$: good enough for most applications
- Limited variables for comparison with observations (L, R, M, stellar type, event – e.g. SN or nova – rates; also $\dot{M}$)
- But this is enough to ID many types of binaries e.g. X-ray binaries, symbiotic stars, double degenerate pairs, Algols etc. and constrain (some) free parameters in the physics

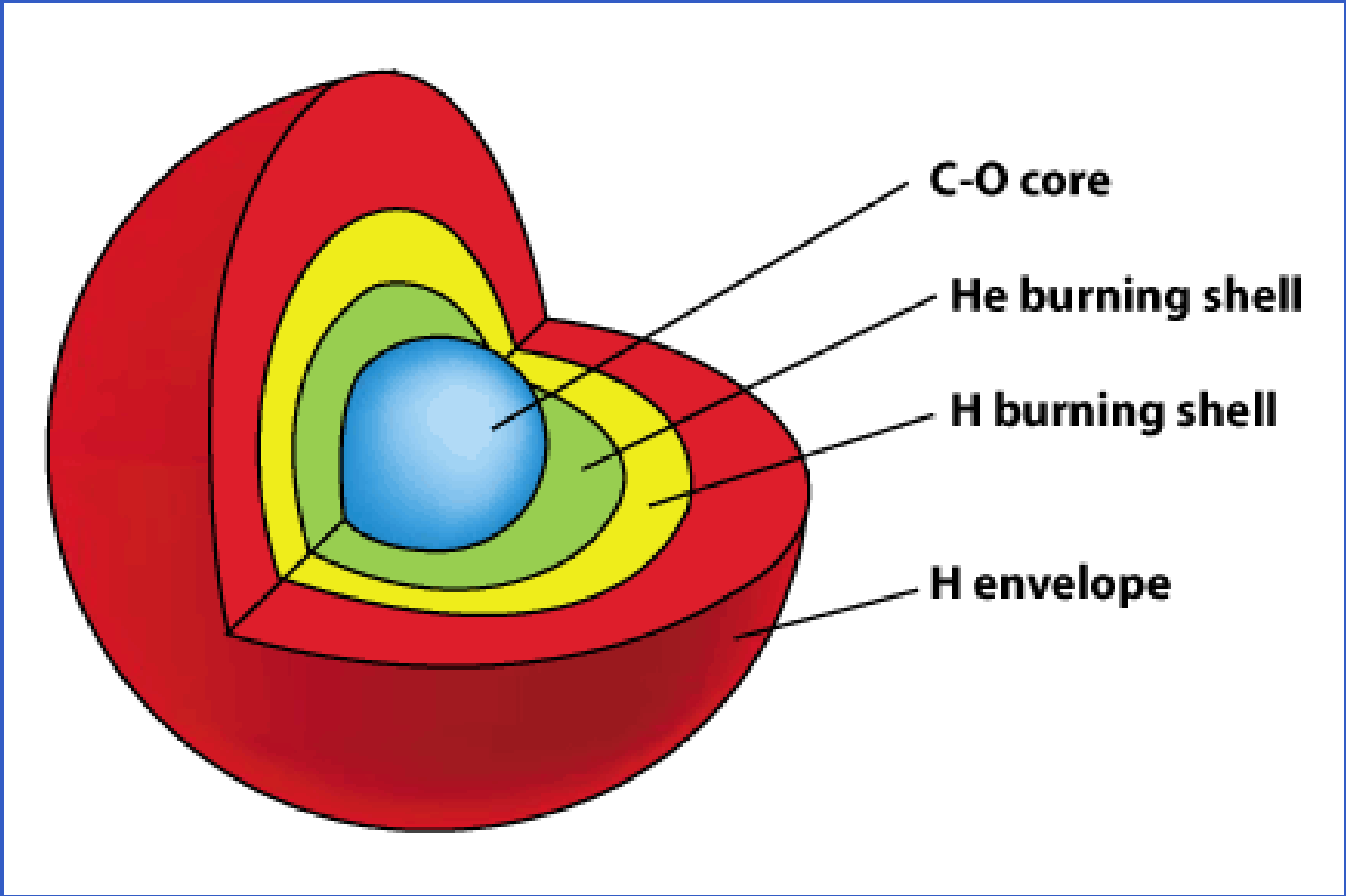
Population Nucleosynthesis

- Introduce nucleosynthesis into synthetic model
- Comparison variable set extends to L, R, M, $\dot{M}$, stellar type, event rate and surface abundances of more than 130 isotopes.
- Provides extra constraints on the models' free parameters.
- Fast/accurate nucleosynthesis model (observations $\sigma \sim 0.1$ dex)
- Synthetic AGB (Iben, Renzini, Groenewegen, Forestini etc)

Nucleosynthesis in Stars



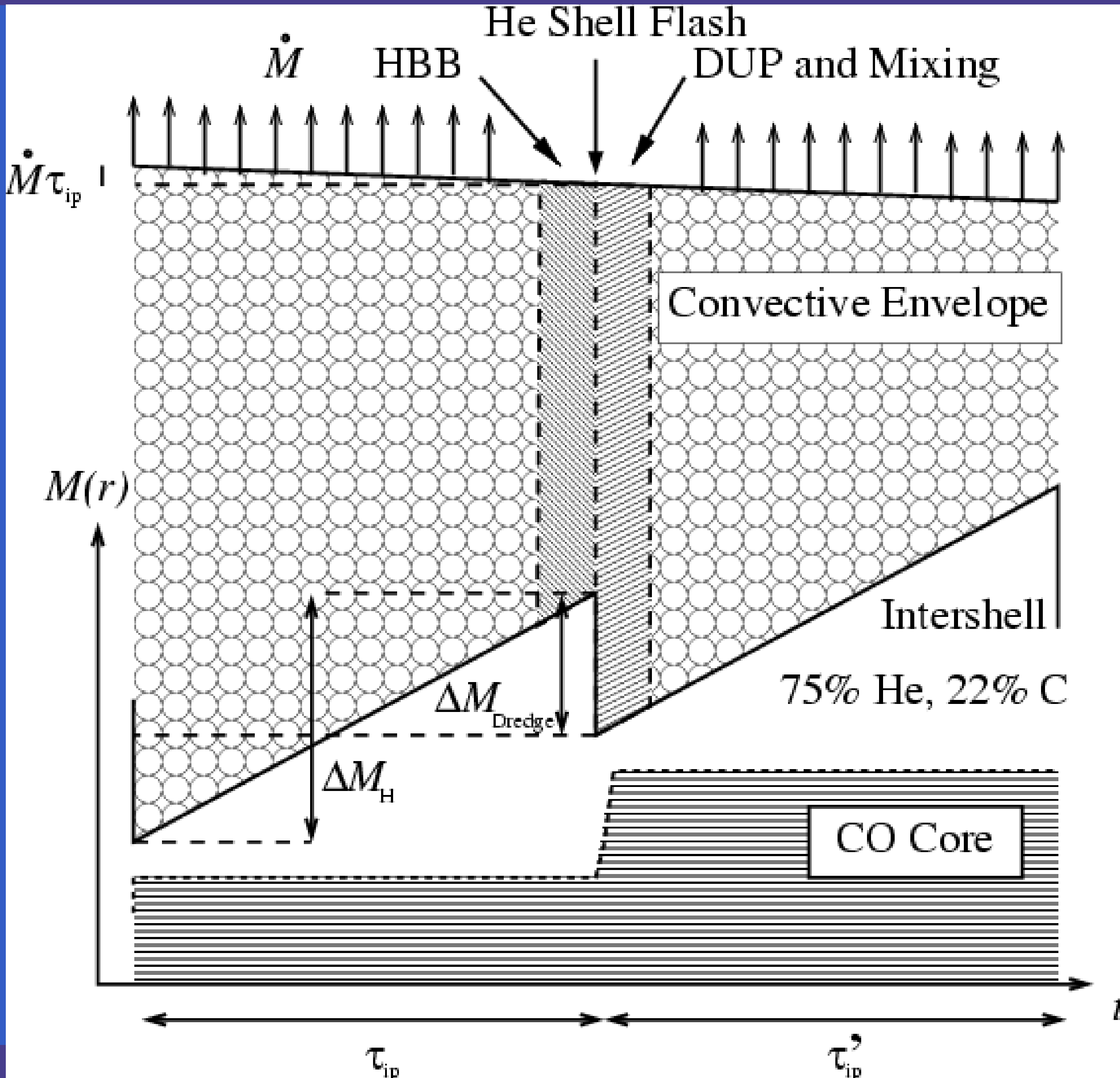
TPAGB star



Nucleosynthesis in TPAGB Stars

- Helium burning converts ^{4}He into ^{12}C, ^{16}O and ^{20}Ne during each pulse (more on this later!)
- The convective hydrogen envelope mixes into the helium-burnt region
- Carbon, oxygen and neon are brought to the surface during each pulse
- This is the “Third Dredge Up”
- NB Difficult to model! Need high resolution, tough numerics.

Thermal Pulses

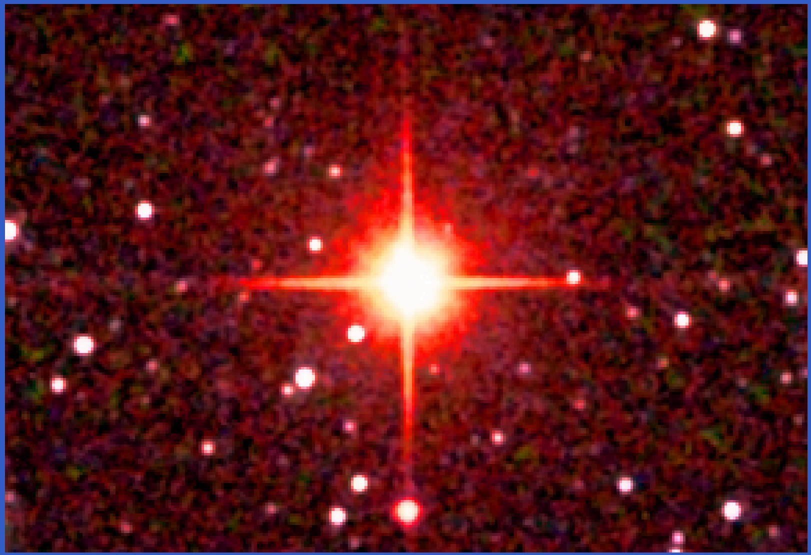


Third Dredge Up

- Occurs when core mass M_{c} exceeds a threshold $M_{\mathrm{c,min}}$
- Amount of material mixed: $\lambda = \Delta M_{\mathrm{Dredge}} / \Delta M_{\mathrm{H}}$
- Include in models by fitting the parameters λ and $M_{\mathrm{c,min}}$
- Fit intershell abundances to detailed models: assume $f(M, Z)$

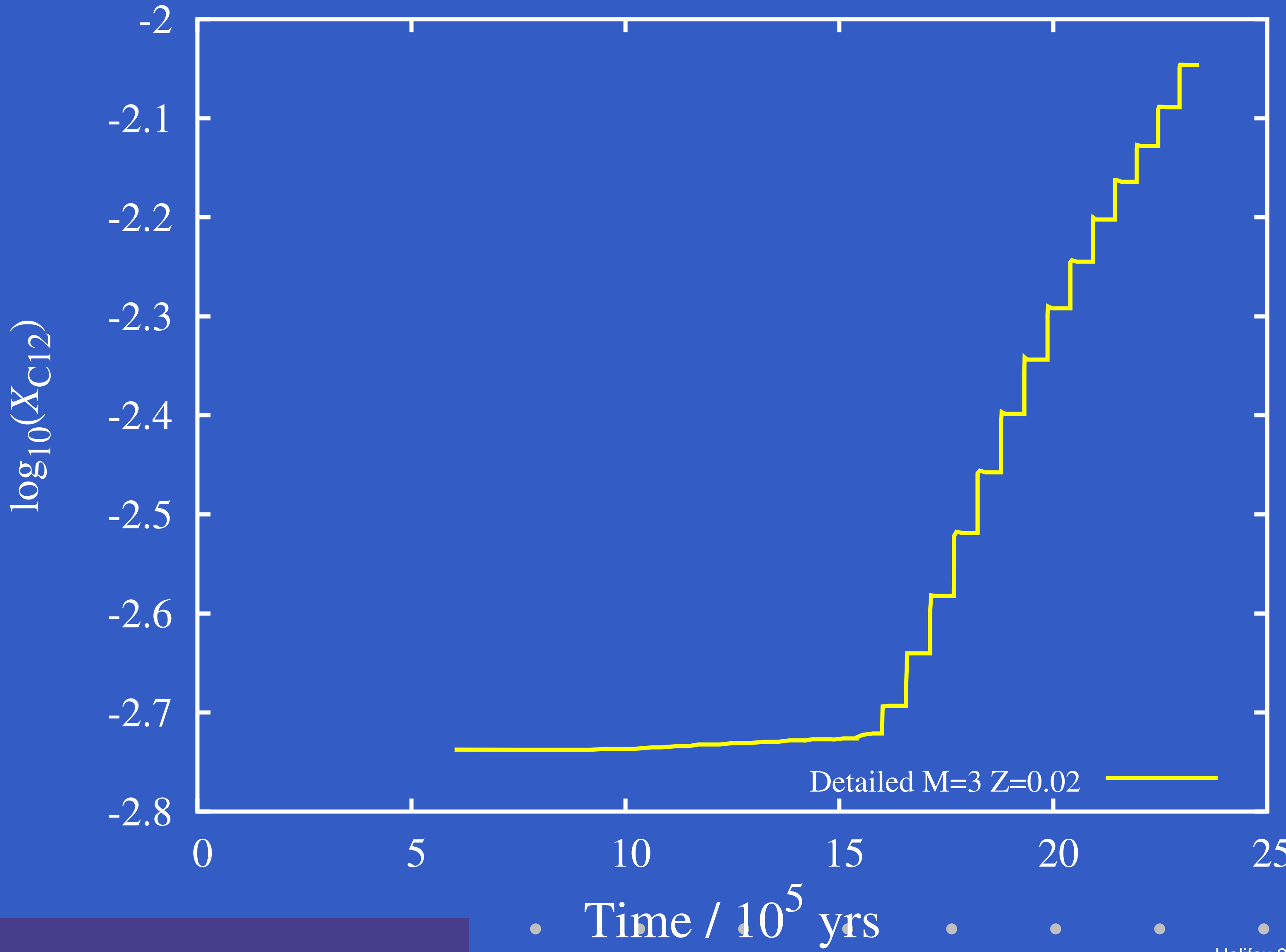
Carbon Stars

- Surface carbon increases: star becomes a "Carbon Star"
- Easily visible (bright, ID by photometry) and complete SMC/LMC surveys exist



V713 Monocerotis

Carbon DUP

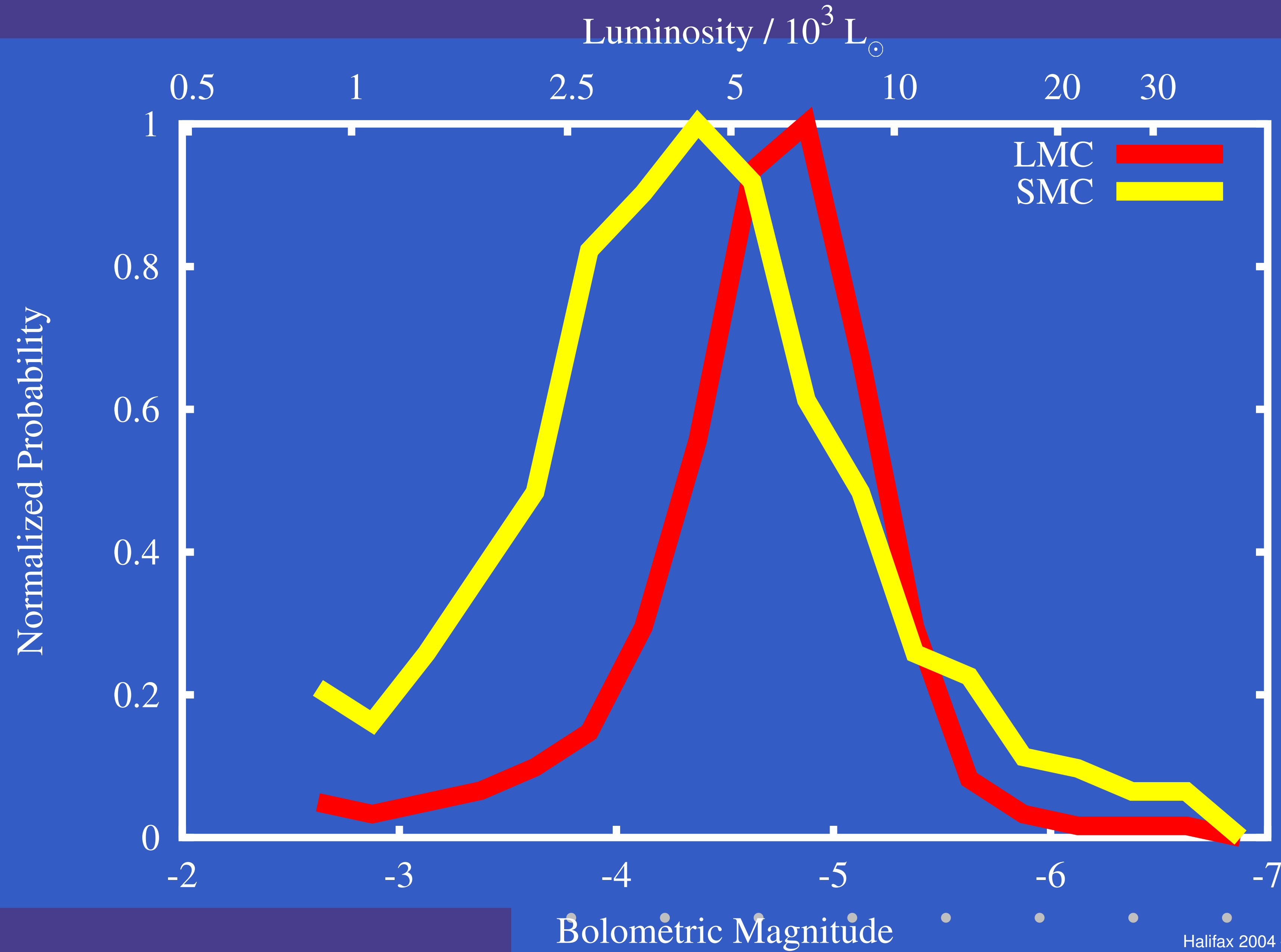


-2
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
-2.8
0
5
10
15
20
25
$\log_{10}(X_{C12})$
Time / 10^5 yrs
Detailed M=3 Z=0.02

Calibrating λ and $M_{\text{c,min}}$

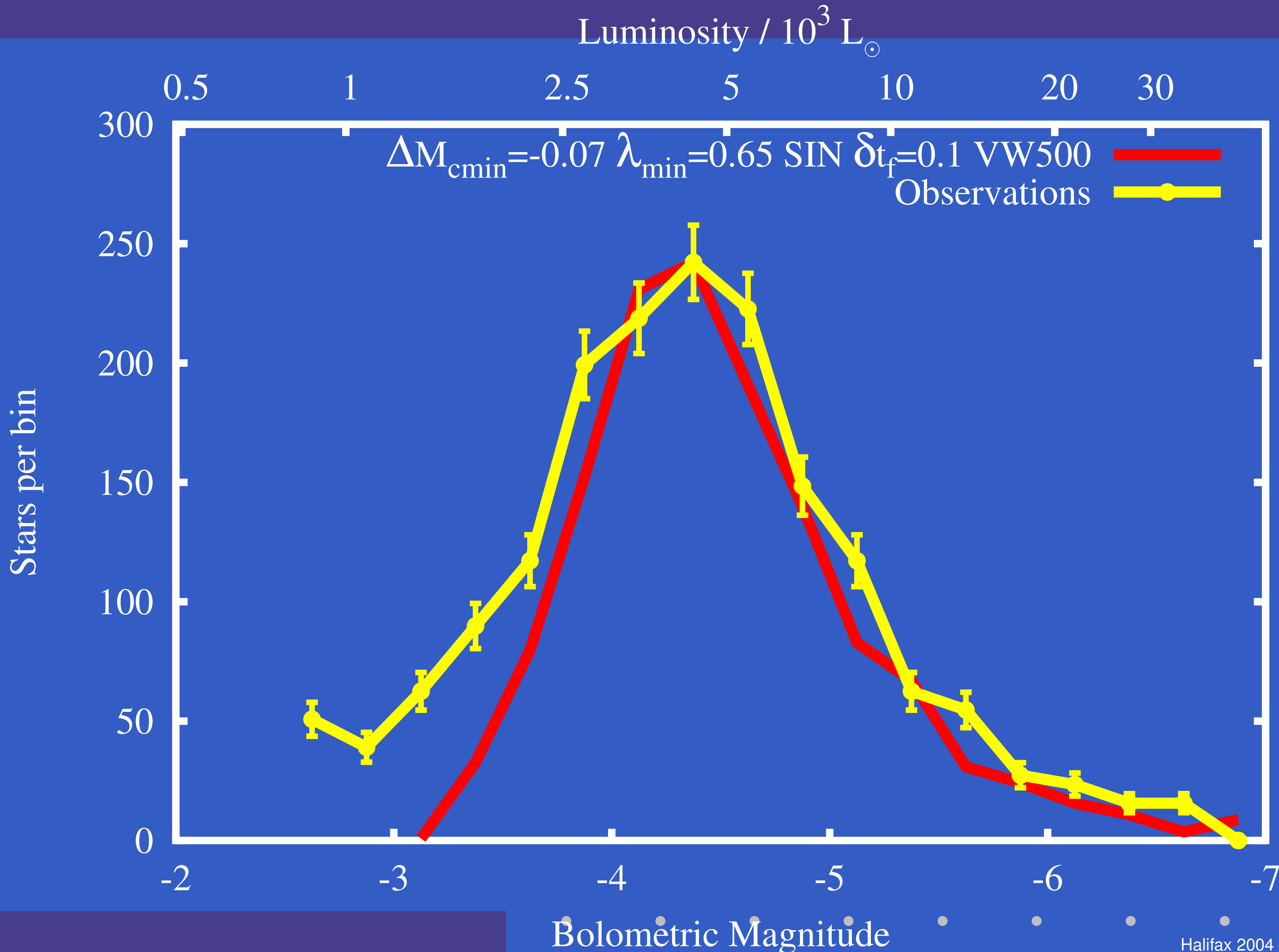
- λ and $M_{\text{c,min}}$ are fitted to Amanda's detailed models: Are they correct?
- Construct the luminosity function of carbon stars by modelling a population
- Define λ_{min} such that $\lambda = \max\left(\lambda_{\text{min}},\ \lambda_{\text{fit}}\right)$
- Define $\Delta M_{\text{c,min}}$ so $M_{\text{c,min}} = M_{\text{c,min}}^{\text{fit}} + \Delta M_{\text{c,min}}$
- Plot $dN/d\text{mag}$ for different λ_{min} and $\Delta M_{\text{c,min}}$
- Use χ^2 test: best fit to LMC ($Z = 0.008$) and SMC ($Z = 0.004$).

CSLFs



Luminosity / 10^3 $L_\odot$
0.5
1
2.5
5
10
20
30
1
0.8
0.6
0.4
0.2
0
Normalized Probability
LMC
SMC
-2
-3
-4
-5
-6
-7
Bolometric Magnitude

Best Fit for the SMC



Best Fit for the SMC

- $\lambda_{\rm min} = 0.65$ and $\Delta M_{\rm c,min} = -0.07\,{\rm M}_\odot$.
- So third dredge-up occurs *earlier* and is *more efficient* than the detailed models predict.
- New (detailed) models by Richard Stancliffe can fit the LMC models nicely.
- But they still fail for the SMC.
- The models are still wrong!
- This is why synthetic models are good.

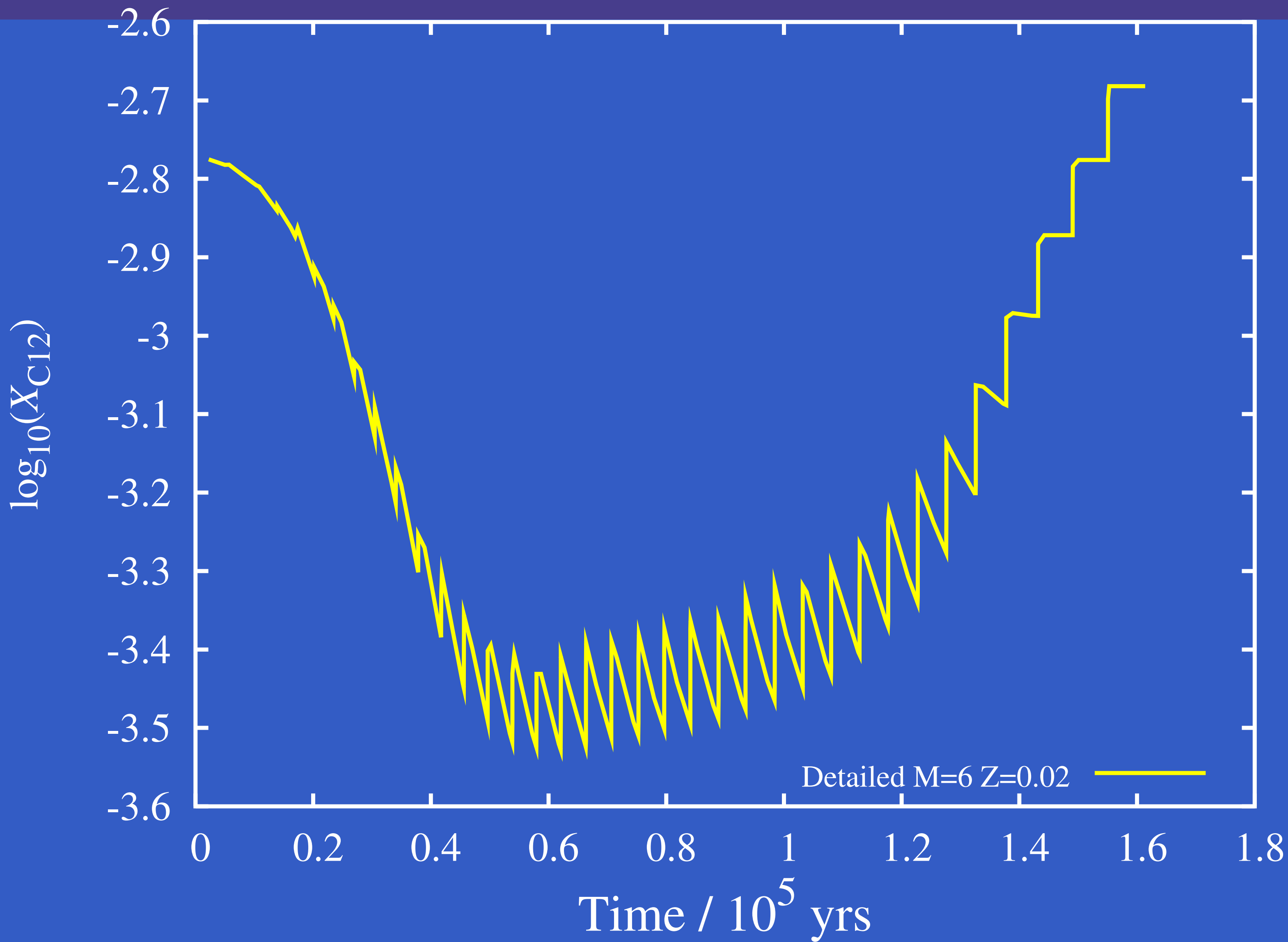
Hot Bottoms?


PATAK'S
PATAK'S
Original
Tastes of INDIA
VINDALOO
CURRY PASTE
CONCENTRATE FOR SAUCES
CUMIN & CHILE
HOT

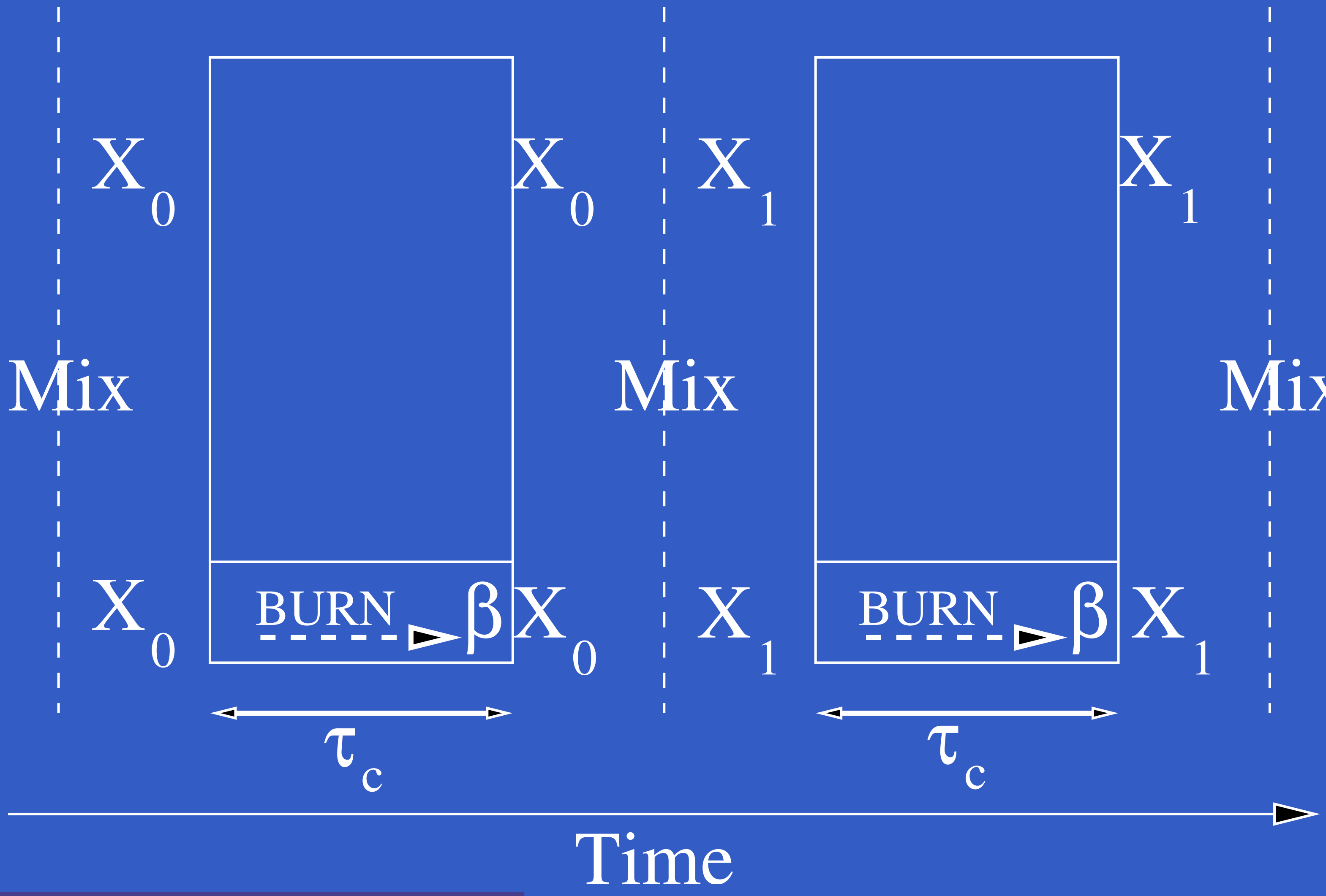
Hot Bottoms

- In stars with $M \gtrsim 4\,\mathrm{M}_\odot$ the base of the convective envelope is hot enough that *Hot Bottom Burning* occurs
- Hydrogen burning occurs in the convective region!
- CNO, NeNa and MgAl cycles may operate, depending on the temperature
- Higher mass $\longrightarrow$ higher temperature
- CNO cycling prevents a star from evolving to a carbon star

Not a Carbon Star



Quick Analytic Burning



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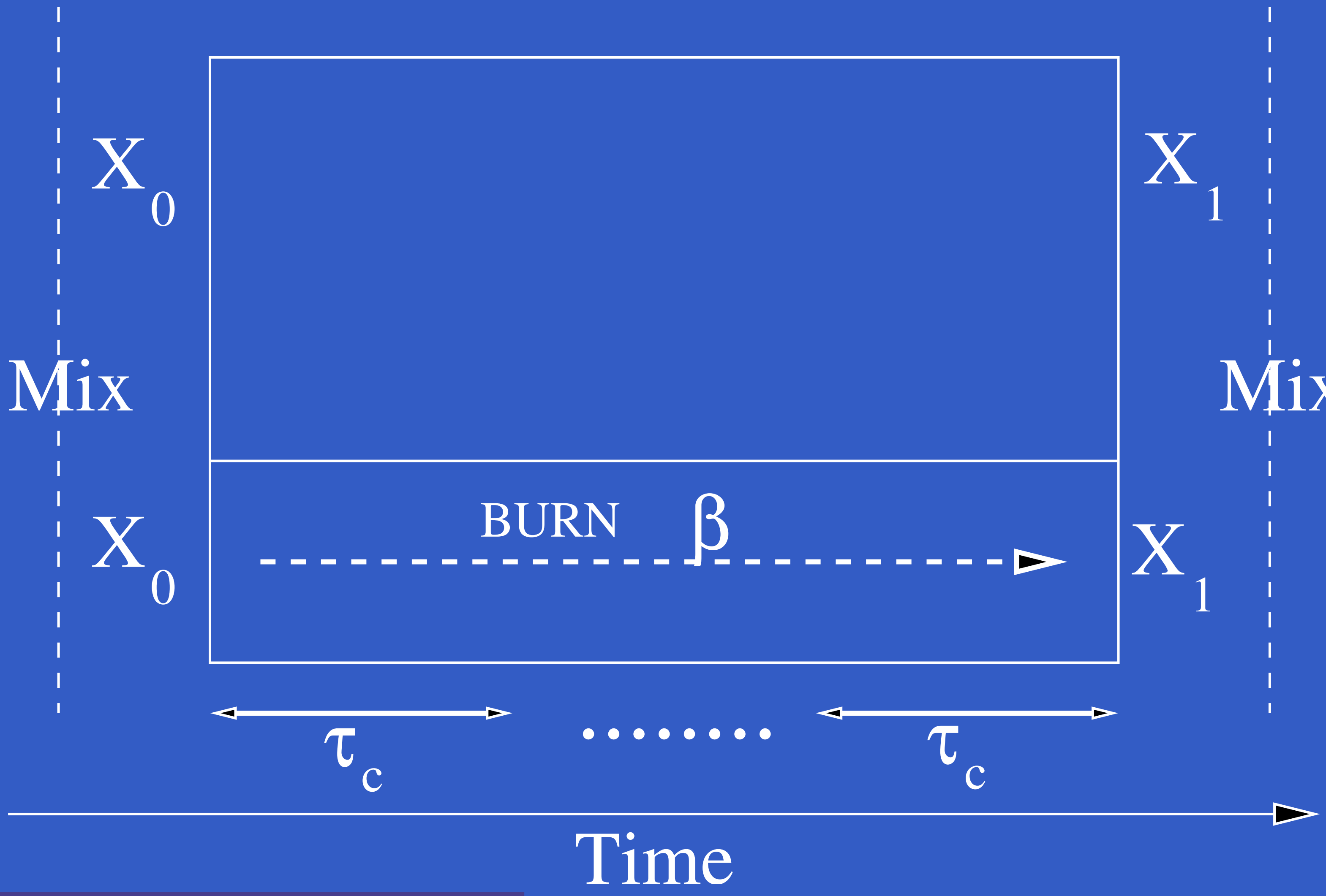


Quick Analytic Burning

- burn $\longrightarrow$ mix $\longrightarrow$ burn $\longrightarrow$ mix. . . replaced by
- A single burn $\longrightarrow$ mix step
- Calibrate amount of burning and burn time to Amanda's detailed models
- Require quick, but accurate, solution to nuclear burning because differential equation solving takes too long
- Iterative method is slow, consider analytic solution

The CNO cycle

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + \alpha \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{16}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu \\
{}^{16}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} &\rightarrow {}^{17}\mathrm{O} + e^{+} + \nu \\
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He} \\
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{18}\mathrm{F} + \gamma \\
{}^{18}\mathrm{F} &\rightarrow {}^{18}\mathrm{O} + e^{+} + \nu \\
{}^{18}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{N} + {}^{4}\mathrm{He}
\end{aligned}
$$

The CN cycle

- $^{15}\text{N} + ^{1}\text{H} \rightarrow ^{16}\text{O} + \gamma$ is slow at low temperature (or short burn times)
- CNO splits to CN and ON cycles
- Express CN cycle as eigenvalue problem

$$\frac{d}{dt}\begin{bmatrix} ^{12}\text{C} \\ ^{13}\text{C} \\ ^{14}\text{N} \end{bmatrix} = \begin{bmatrix} -1/\tau_{12} & 0 & 1/\tau_{14} \\ 1/\tau_{12} & -1/\tau_{13} & 0 \\ 0 & 1/\tau_{13} & -1/\tau_{14} \end{bmatrix} \begin{bmatrix} ^{12}\text{C} \\ ^{13}\text{C} \\ ^{14}\text{N} \end{bmatrix}$$

- Solve for ^{12,13}C, ^{14}N as (simple) $f(t)$.

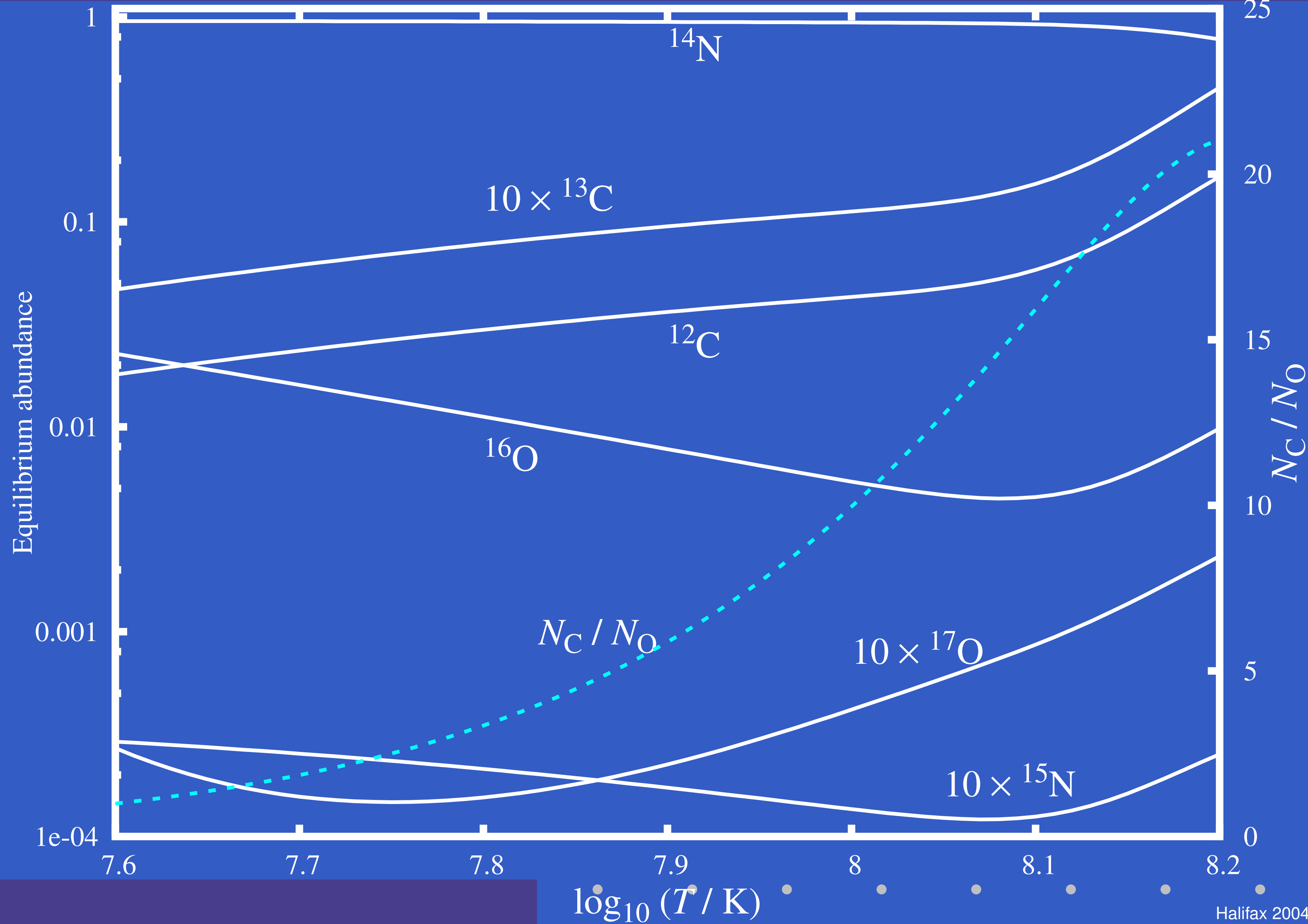
The ON Cycle

- Long burn times: CN cycle $\rightarrow$ eq. $\sim 98\%$ ^{14}N
- ON cycle activates

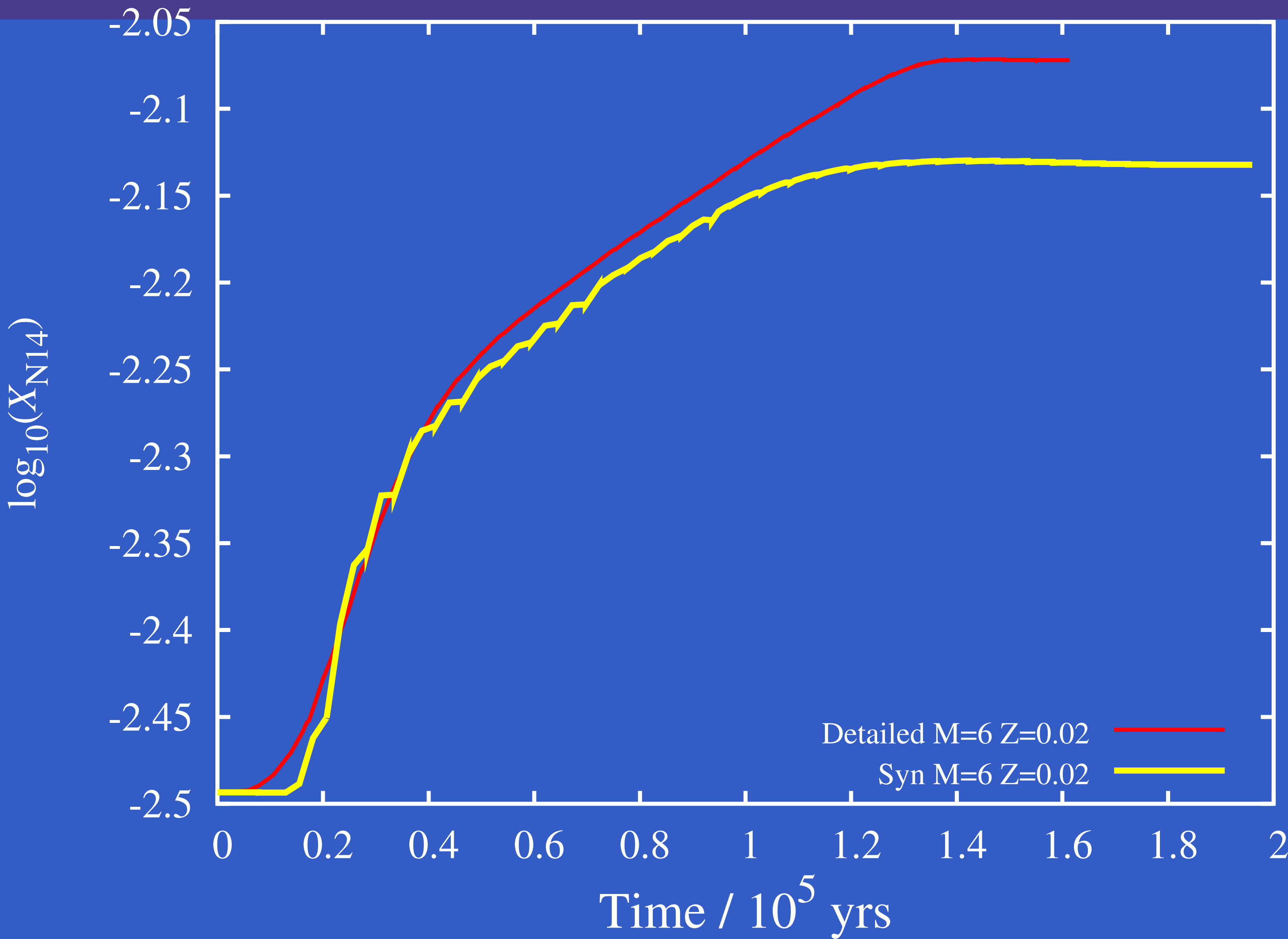
$$\frac{d}{dt}\begin{bmatrix} ^{14}\mathrm{N} \\ ^{16}\mathrm{O} \\ ^{17}\mathrm{O} \end{bmatrix} = \begin{bmatrix} -1/\tau_{12} & 0 & 1/\tau_{14} \\ 1/\tau_{12} & -1/\tau_{13} & 0 \\ 0 & 1/\tau_{13} & -1/\tau_{14} \end{bmatrix}\begin{bmatrix} ^{14}\mathrm{N} \\ ^{16}\mathrm{O} \\ ^{17}\mathrm{O} \end{bmatrix}$$

- Oxygen (slowly!) destroyed.
- Again ^{14}N is the result.

CNO Equilibrium



Synthetic vs Detailed



NeNa cycle

$$
\begin{array}{ccccccc}
 & & {}^{21}\mathrm{Ne} & \xrightarrow{(p,\gamma)} & {}^{22}\mathrm{Na} & & \\
 & & (\beta^+)\uparrow & & \downarrow(\beta^+) & & \\
 & & {}^{21}\mathrm{Na} & & {}^{22}\mathrm{Ne} & & \\
 & & (p,\gamma)\uparrow & & \downarrow(p,\gamma) & & \\
{}^{19}\mathrm{F} & \xrightarrow{(p,\gamma)} & {}^{20}\mathrm{Ne} & \xleftarrow{(p,\alpha)} & {}^{23}\mathrm{Na} & \xrightarrow{(p,\gamma)} & \mathrm{MgAl}
\end{array}
$$

Simpler NeNa cycle

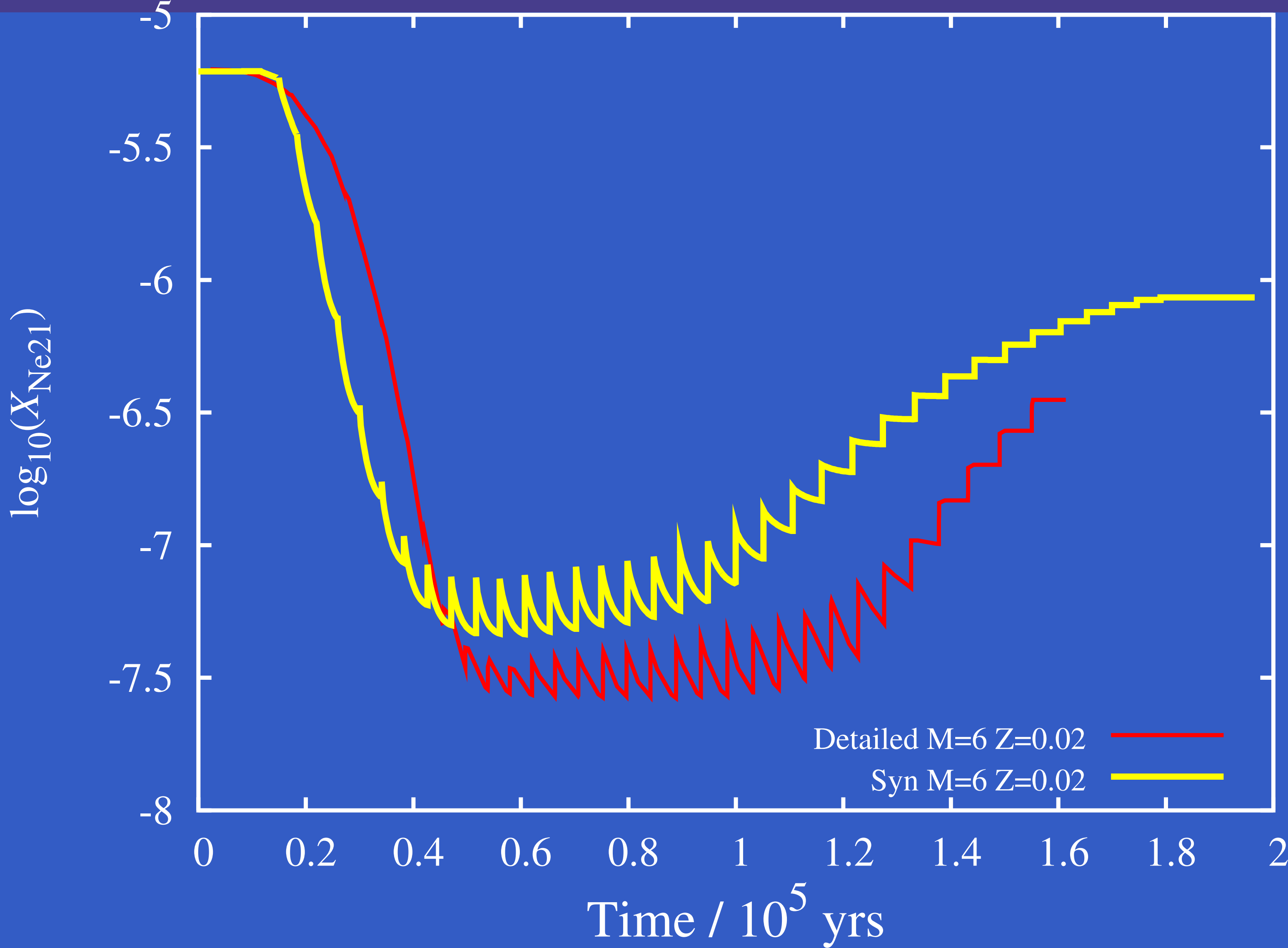
$$\begin{array}{ccc} {}^{21}\mathrm{Ne} & \xrightarrow{(p,\gamma)} & {}^{22}\mathrm{Ne} \\ (p,\gamma)\uparrow & & \downarrow(p,\gamma) \\ {}^{20}\mathrm{Ne} & \xleftarrow{(p,\alpha)} & {}^{23}\mathrm{Na} \end{array}$$

Another eigenvalue problem

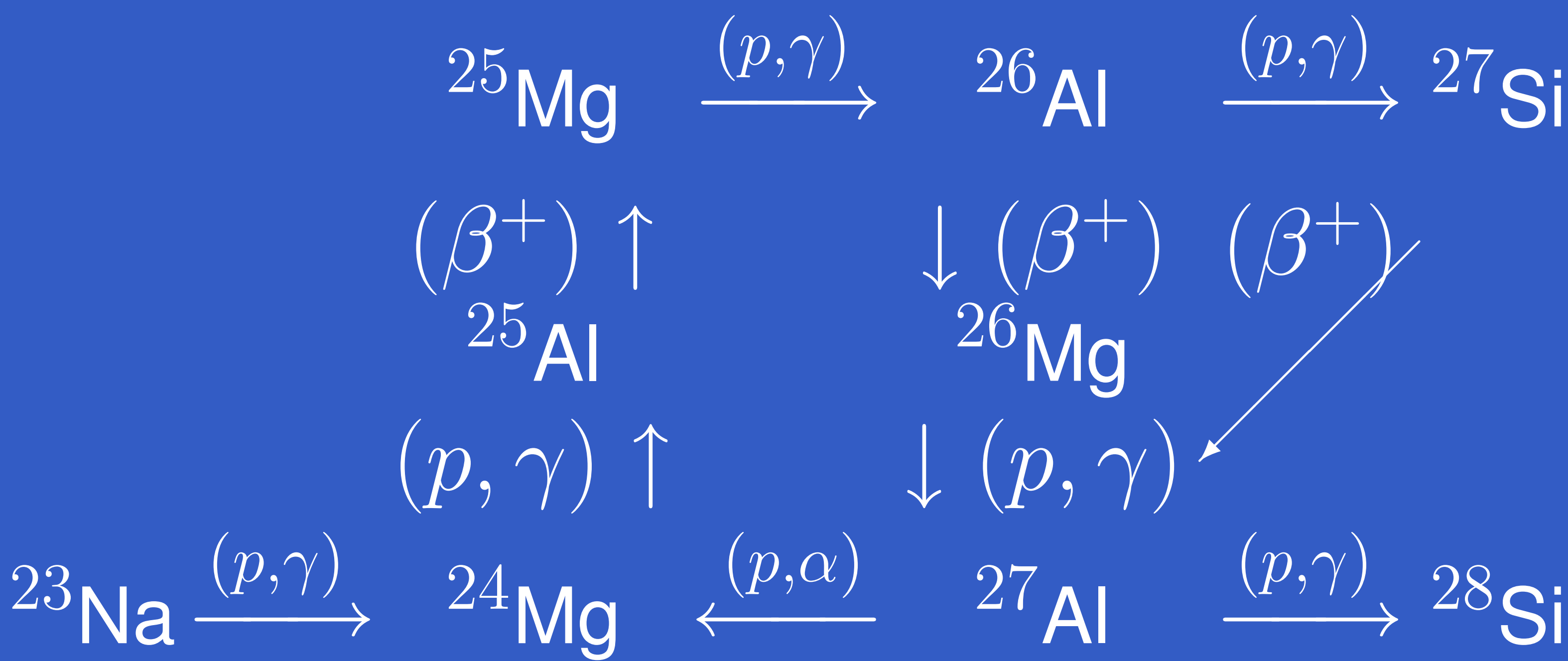
$$\frac{d}{dt}\begin{bmatrix} ^{20}\mathrm{Ne} \\ ^{21}\mathrm{Ne} \\ ^{22}\mathrm{Ne} \\ ^{23}\mathrm{Na} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_{20}} & 0 & 0 & \frac{1}{\tau_{23}} \\ \frac{1}{\tau_{20}} & \frac{1}{-\tau_{21}} & 0 & 0 \\ 0 & \frac{1}{\tau_{21}} & -\frac{1}{\tau_{22}} & 0 \\ 0 & 0 & \frac{1}{\tau_{22}} & -\frac{1}{\tau_{23}} \end{bmatrix} \begin{bmatrix} ^{20}\mathrm{Ne} \\ ^{21}\mathrm{Ne} \\ ^{22}\mathrm{Ne} \\ ^{23}\mathrm{Na} \end{bmatrix}$$

$\rightarrow$analytic solution. . .

Example: ^{21}Ne



MgAl cycle. . .

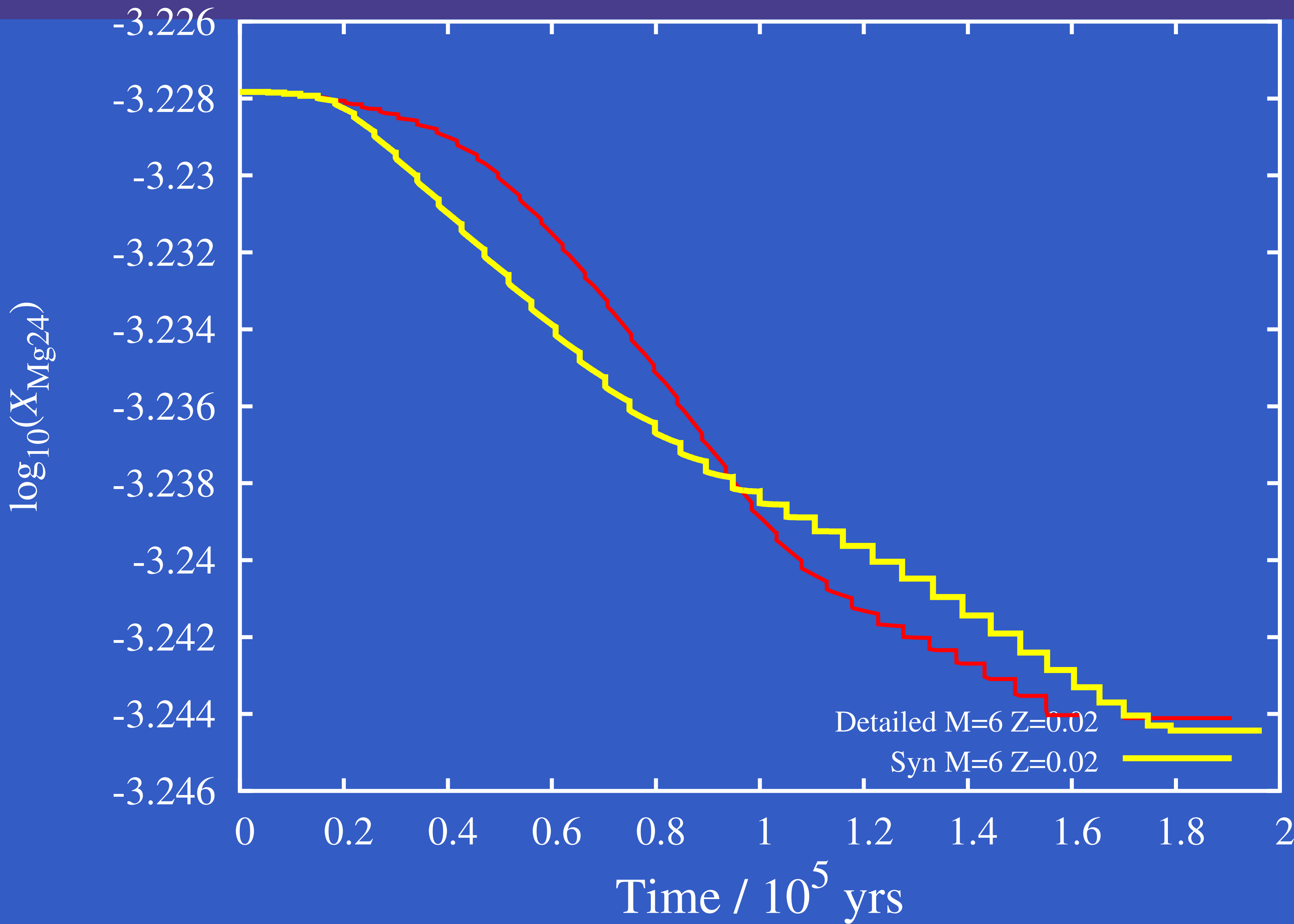


^{25}Mg
(p,γ)
^{26}Al
(p,γ)
^{27}Si
(β+)
(β+)
(β+)
^{25}Al
^{26}Mg
(p, γ)
(p, γ)
^{23}Na
(p,γ)
^{24}Mg
(p,α)
^{27}Al
(p,γ)
^{28}Si

Becomes the MgAl Chain

- $$\frac{d^{24}\text{Mg}}{dt} = -\frac{^{24}\text{Mg}}{\tau_{24}},$$
- $$\frac{d^{25}\text{Mg}}{dt} = -\frac{^{25}\text{Mg}}{\tau_{25}} + \frac{^{24}\text{Mg}}{\tau_{24}},$$
- $$\frac{d^{26}\text{Al}}{dt} = \frac{^{25}\text{Mg}}{\tau_{25}} - \frac{^{26}\text{Al}}{\tau_{\beta 26}} - \frac{^{26}\text{Al}}{\tau_{26'}} = \frac{^{25}\text{Mg}}{\tau_{25}} - \frac{^{26}\text{Al}}{\tau_{26}'},$$
- $$\frac{d^{26}\text{Mg}}{dt} = \frac{^{26}\text{Al}}{\tau_{\beta 26}} - \frac{^{26}\text{Mg}}{\tau_{26}}$$
- $$\frac{d^{27}\text{Al}}{dt} = \frac{^{26}\text{Mg}}{\tau_{26}} + \frac{^{26}\text{Al}}{\tau_{26'}} > 0$$

Example: ^{24}Mg



Chemical Yields

- Galactic Chemical Evolution models require calculations of *yields*
- This is the amount of mass ejected as each isotope from a population of stars
- Synthetic models can be used to calculate the yields from single *and binary stars*
- Uncertainties due to variable initial distributions / physics

Chemical Yields

One definition (my definition)

$$\text{yield of } i = \frac{\text{MASS OUT AS ISOTOPE } i}{\text{MASS INTO STARS}}$$

- Mass into stars is different for single and binary populations, this definition takes that into account.
- Binary interaction reduces number of GB and AGB stars, so reduces the yield of isotopes produced in GB and AGB stars.

Nitrogen Yield (Integrated)

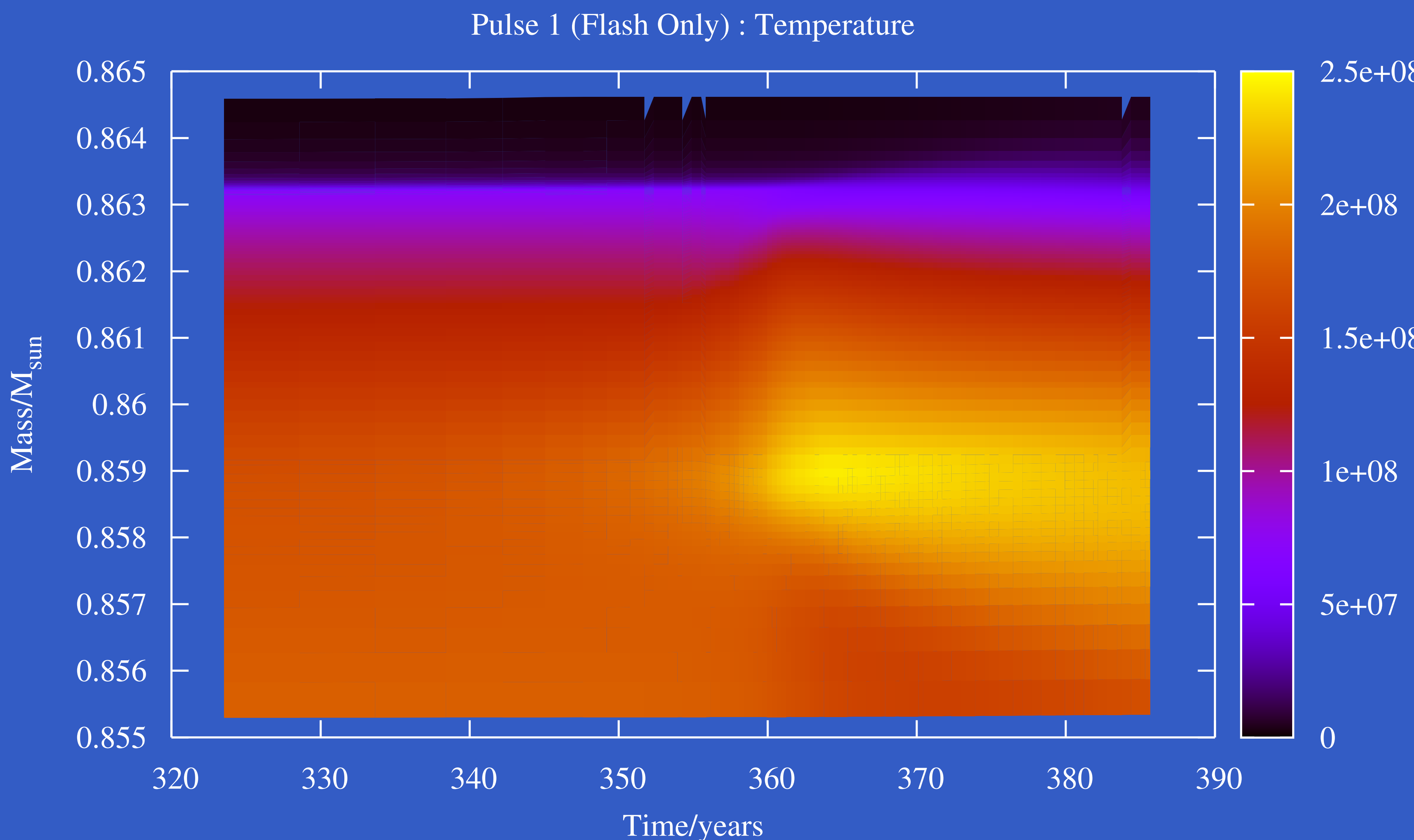
Integrated nitrogen yield (mass out as ^{14}N / mass input to stars)

- Single Stars 1.294×10^{-3}
- Binary Stars 9.878×10^{-4}
- Difference due to binaries : -24%

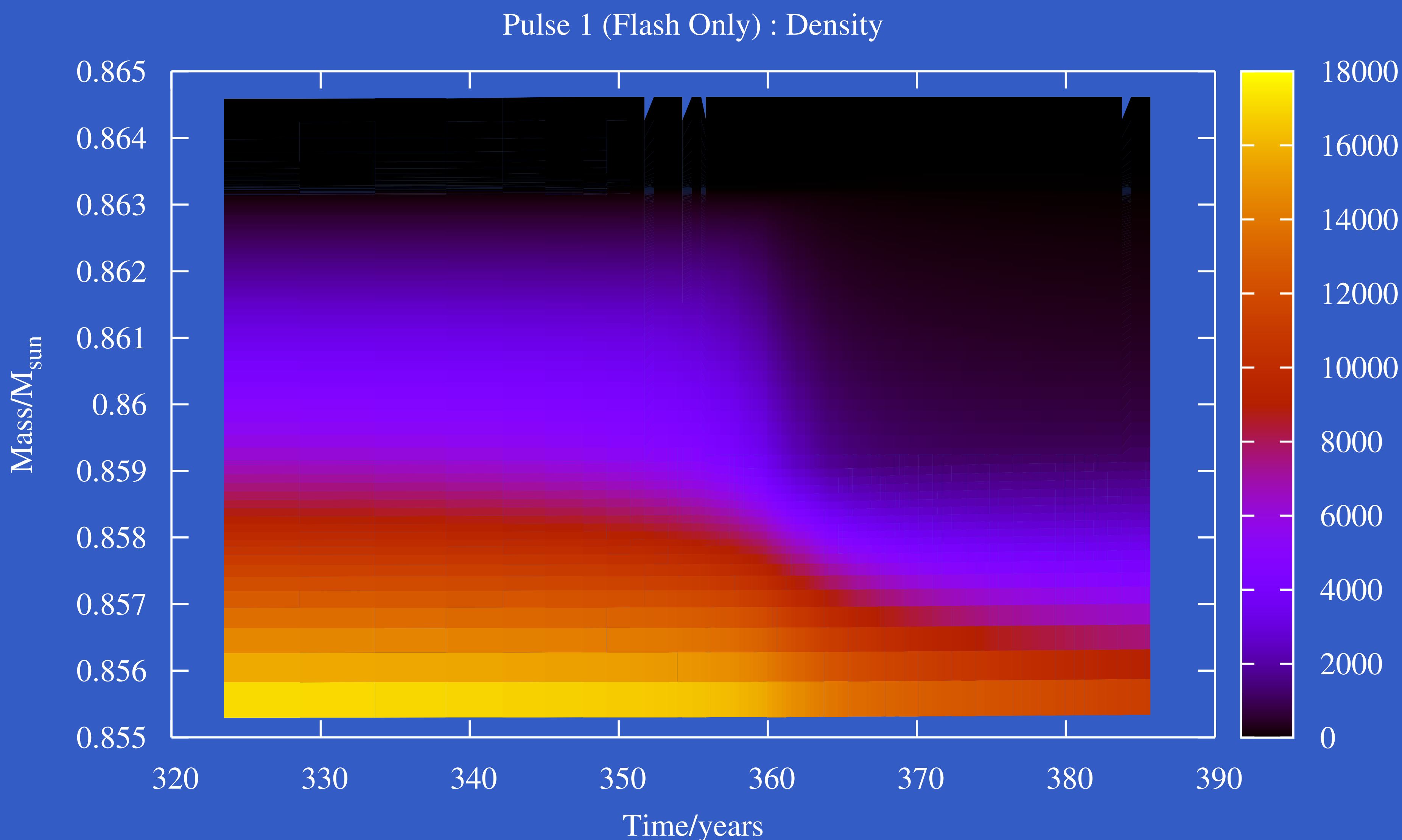
Reaction Rates in the Intershell

- Intershell composition $\rightarrow$ envelope pollution
- Reaction rates determine amount of isotopic production
- Many are quite uncertain (e.g. ^{25}Mg $(\alpha, n)^{28}$Si fac. 10^5!)
- Can we quantify the uncertainty in the yields?
- Difficult problem: many rates/stars, takes too long
- Perhaps with a synthetic model?
- Synthetic Helium Burning!
- Convective intershell: much like HBB.

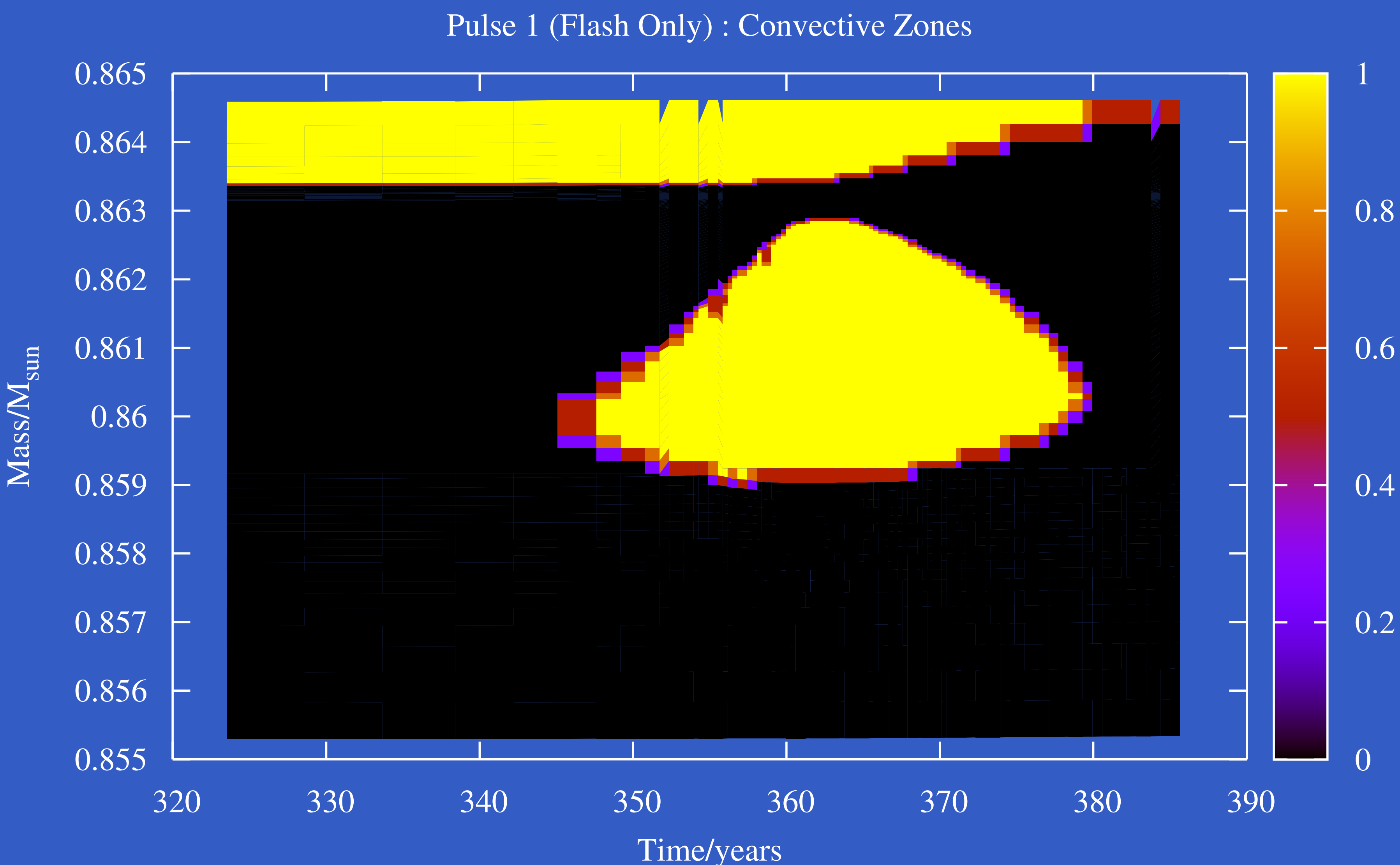
Helium Flash: T



Helium Flash: ρ



Helium Flash: convection



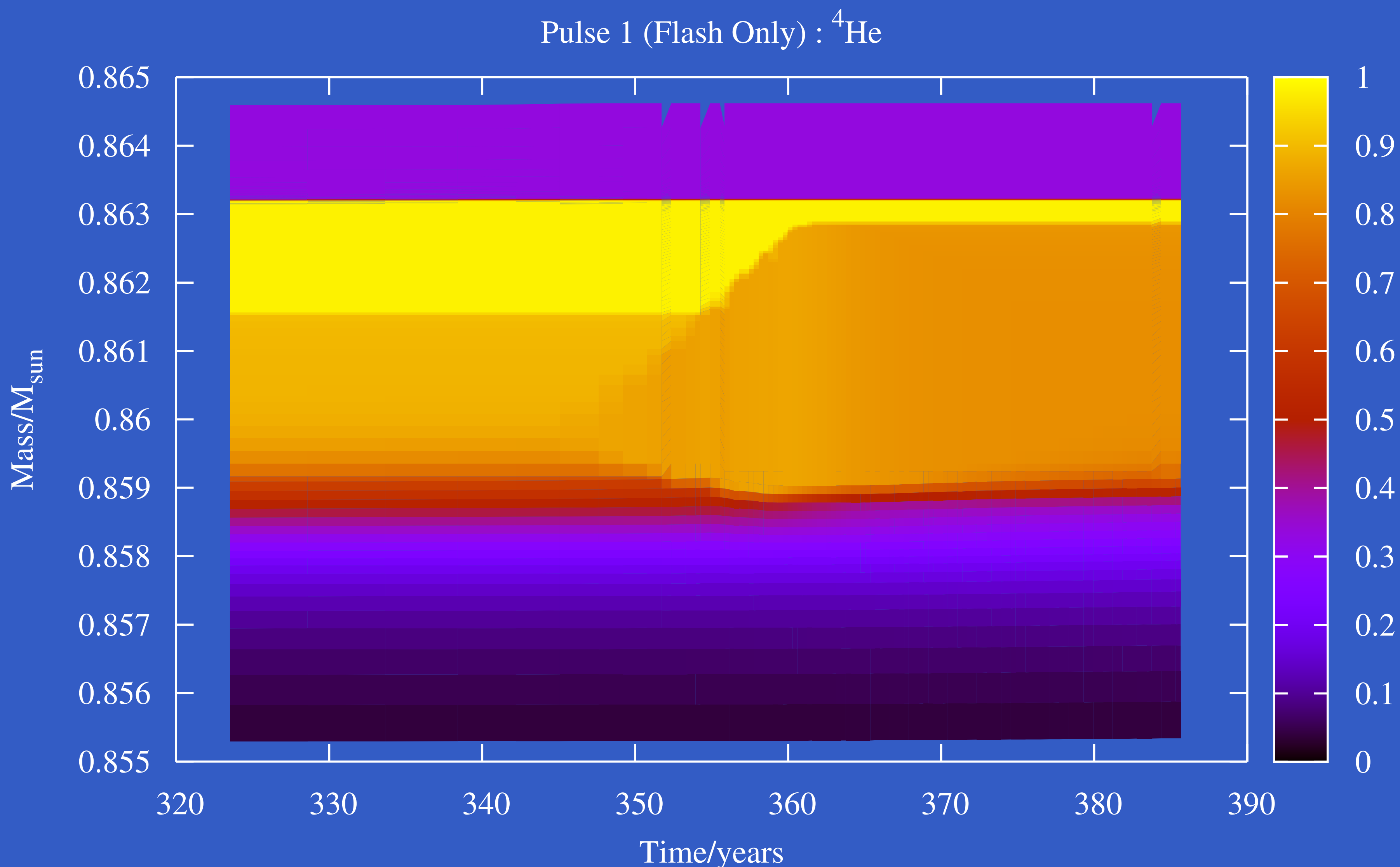
Helium Burning Reactions

Material is processed by complete hydrogen burning, so is 0% ^{1}H, $\sim 98\%$ ^{4}He, CNO $\rightarrow$ ^{14}N, other trace metals (perhaps NeNa/MgAl cycled). Two reaction sets operate

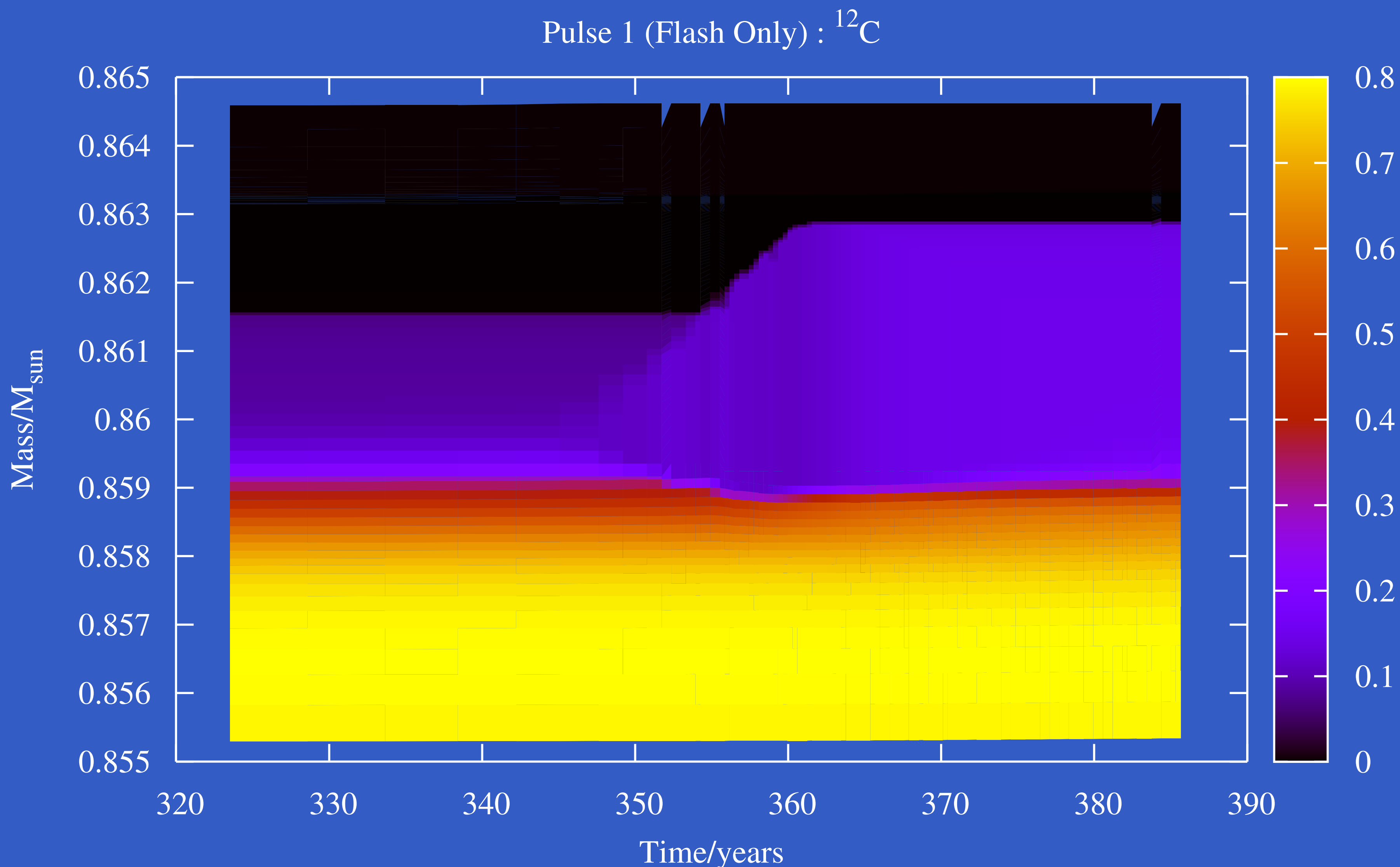
- α-capture e.g. $^{4}\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$, $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$, $^{16}\mathrm{O}(\alpha,\gamma)^{20}\mathrm{Ne}$ etc.
- n-capture e.g. $^{20}\mathrm{Ne}(n,\gamma)^{21}\mathrm{Ne}$, $^{24}\mathrm{Mg}(n,\gamma)^{25}\mathrm{Mg}$, $^{28}\mathrm{Si}(n,\gamma)^{29}\mathrm{Si}$

Detailed models give final abundances $^{4}\mathrm{He} = 0.7$, $^{12}\mathrm{C} = 0.26$, $^{16}\mathrm{O} = 0.004$.

Helium Flash: ^{4}He



Helium Flash: ^{12}C



α captures

$$\frac{d^4\mathrm{He}}{dt} = -3\,\langle\sigma v\rangle_{3\alpha}\left({}^4\mathrm{He}\right)^3 - \langle\sigma v\rangle_{\alpha 12}\,{}^4\mathrm{He}\,{}^{12}\mathrm{C} - \langle\sigma v\rangle_{\alpha 16}\,{}^4\mathrm{He}\,{}^{16}\mathrm{O} - \ldots,$$

$$\frac{d^{12}\mathrm{C}}{dt} = \langle\sigma v\rangle_{3\alpha}\left({}^4\mathrm{He}\right)^3 - \langle\sigma v\rangle_{\alpha 12}\,{}^4\mathrm{He}\,{}^{12}\mathrm{C}\,,$$

$$\frac{d^{16}\mathrm{O}}{dt} = \langle\sigma v\rangle_{\alpha 12}\,{}^4\mathrm{He}\,{}^{12}\mathrm{C} - \langle\sigma v\rangle_{\alpha 16}\,{}^4\mathrm{He}\,{}^{16}\mathrm{O}$$

$$\frac{d^{20}\mathrm{Ne}}{dt} = \langle\sigma v\rangle_{\alpha 16}\,{}^4\mathrm{He}\,{}^{16}\mathrm{O} - \langle\sigma v\rangle_{\alpha 20}\,{}^4\mathrm{He}\,{}^{20}\mathrm{Ne}$$

$$\ldots$$

First approximation...

$$\frac{d^4\mathrm{He}}{dt} = -3\,\langle\sigma v\rangle_{3\alpha}\left({}^4\mathrm{He}\right)^3 - \langle\sigma v\rangle_{\alpha 12}\,{}^4\mathrm{He}\,{}^{12}\mathrm{C} - \langle\sigma v\rangle_{\alpha 16}\,{}^4\mathrm{He}\,{}^{16}\mathrm{O},$$

$$\frac{d^{12}\mathrm{C}}{dt} = \langle\sigma v\rangle_{3\alpha}\left({}^4\mathrm{He}\right)^3 - \langle\sigma v\rangle_{\alpha 12}\,{}^4\mathrm{He}\,{}^{12}\mathrm{C}\,,$$

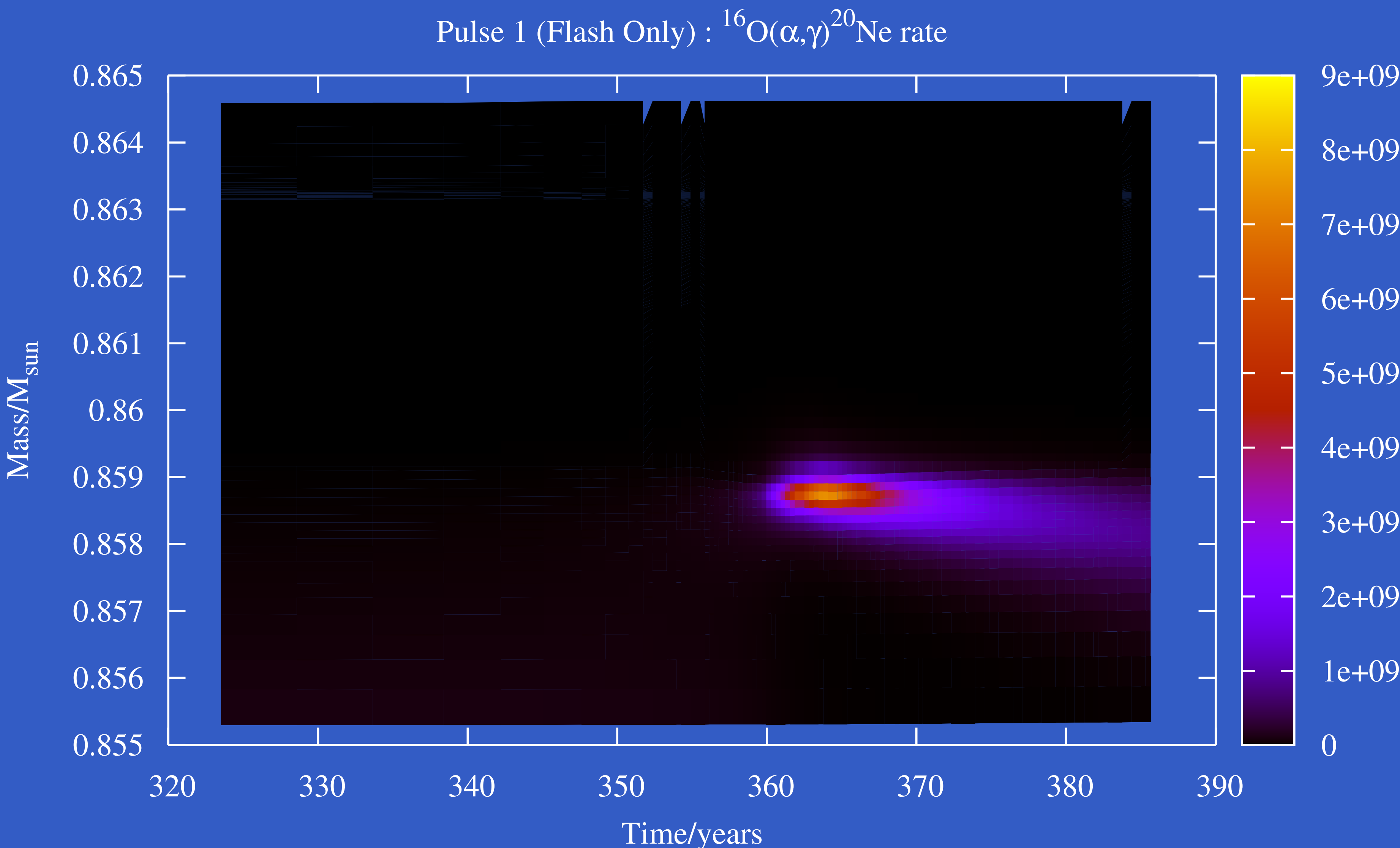
$$\frac{d^{16}\mathrm{O}}{dt} = \langle\sigma v\rangle_{\alpha 12}\,{}^4\mathrm{He}\,{}^{12}\mathrm{C} - \langle\sigma v\rangle_{\alpha 16}\,{}^4\mathrm{He}\,{}^{16}\mathrm{O}$$

$$\frac{d^{20}\mathrm{Ne}}{dt} = \langle\sigma v\rangle_{\alpha 16}\,{}^4\mathrm{He}\,{}^{16}\mathrm{O} - \langle\sigma v\rangle_{\alpha 20}\,{}^4\mathrm{He}\,{}^{20}\mathrm{Ne}$$

But it's not really like HBB!

- Some of the material ingested into the convective pocket is *not from hydrogen burning*
- In fact it is *radiatively He-burnt material!*
- Abundances $^{4}\mathrm{He} \sim 0.55$, $^{12}\mathrm{C} \sim 0.4$, $^{16}\mathrm{O} \sim 0.015$ are a function of mass
- Contribution due to this "dredge up" is small but contributes to the $^{12}\mathrm{C}$, $^{16}\mathrm{O}$, $^{20}\mathrm{Ne}$
- *Not* easy to model with a simple algorithm :(

^{16}O$(\alpha, \gamma)^{20}$Ne Burn Rate



Prototype Model

- Assume constant abundance radiative dredge-up material
- Fit T and ρ (max) of convective pulse vs t
- Burn a fraction of the intershell at T, ρ for the pulse duration (~ 25 years)
- Fit a constant mixing rate (burnt + unburnt) to Amanda's model results

Iterative Burning

- Traditional technique, required because of $^4He^3$ term and significant ^{12}C-burning
- At each timestep use iterative relaxation method to calculate abundances
- n-capture equilibrium
- Code in Perl, easy to experiment or change (or break!)
- slower than C or the evil F so must be efficient
- Iterative solution agrees with Runge-Kutta solution

$M = 5\,\mathrm{M}_\odot$, $Z = 0.02$, **TP19, mix rate 1**

Isotope	Amanda	Rob
^{4}He	0.7007	0.7064
^{12}C	0.2666	0.2656
^{16}O	0.004212	0.002608
^{20}Ne	0.001585	0.001566
^{21}Ne	2.869×10^{-5}	2.328×10^{-5}
^{22}Ne	0.01794	0.01921
^{24}Mg	0.0001174	0.00009838
^{25}Mg	0.002673	0.001504
^{26}Mg	0.003016	0.001613

Why simple HBB model worked

- T and ρ are $\sim$constant over an interpulse period
- Burning shell is thin compared to M_{conv}
- Total amount burned $< M_{\text{conv}}$
- Hydrogen abundance $\sim$ constant over an interpulse

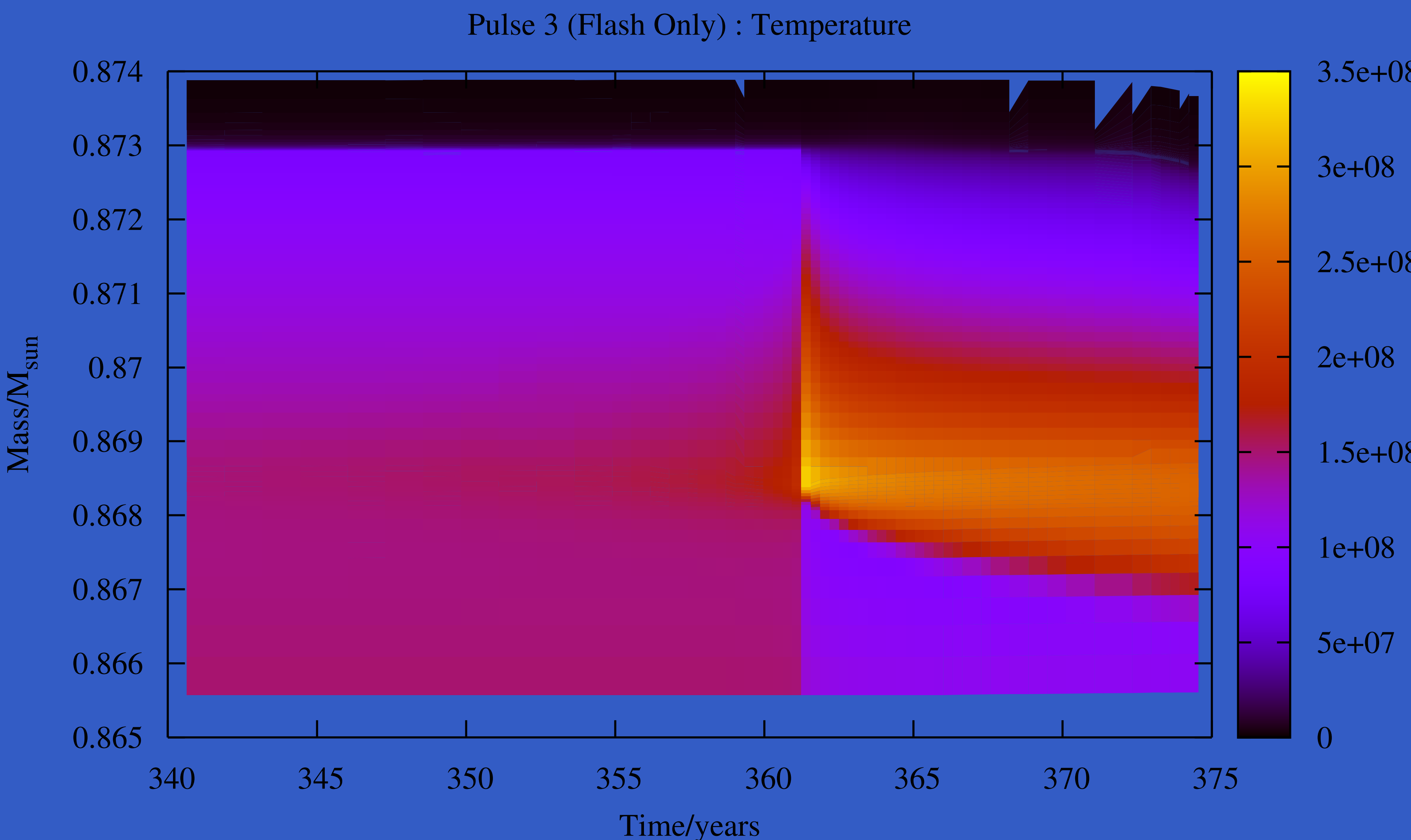
Why simple He-burning model fails

- T and ρ are definitely not constant in mass or time
- Burning shell is not thin compared to M_{conv}?
- Total amount burned $\gg M_{\text{conv}}$
- Helium abundance is not constant
- Mixing rate is not constant?
- *Not* because neutrons are out of eq.
- Burning by iterative network is much slower (but quicker than RK!)

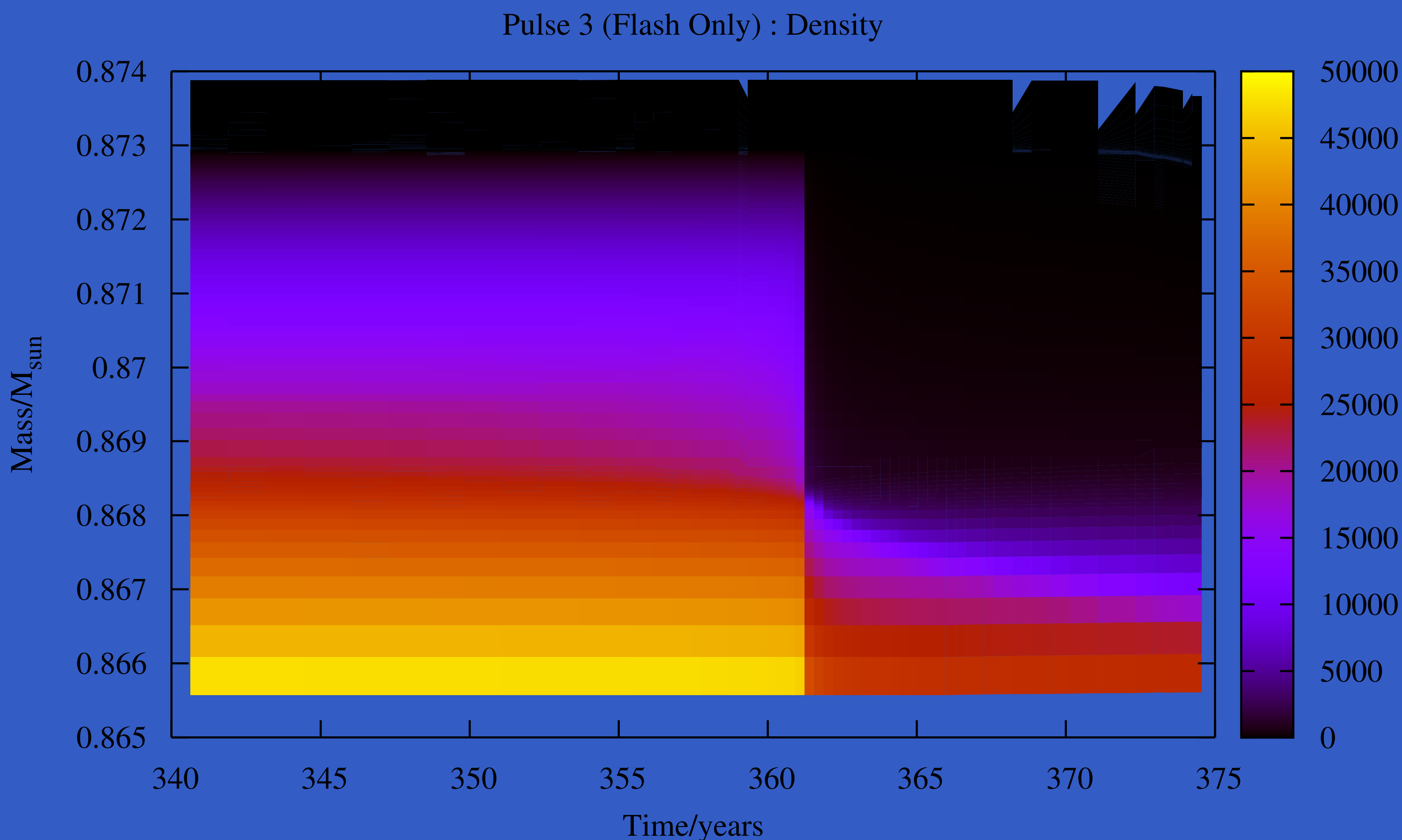
Conclusions

- Need to model radiative burning properly: need mass grid
- Convective burning tricky without T, ρ, Y and mixing as $f(t)$
- We may as well use detailed models. . .
- It's just too complicated for synthetic models.
- Easier to fit results of detailed model runs.

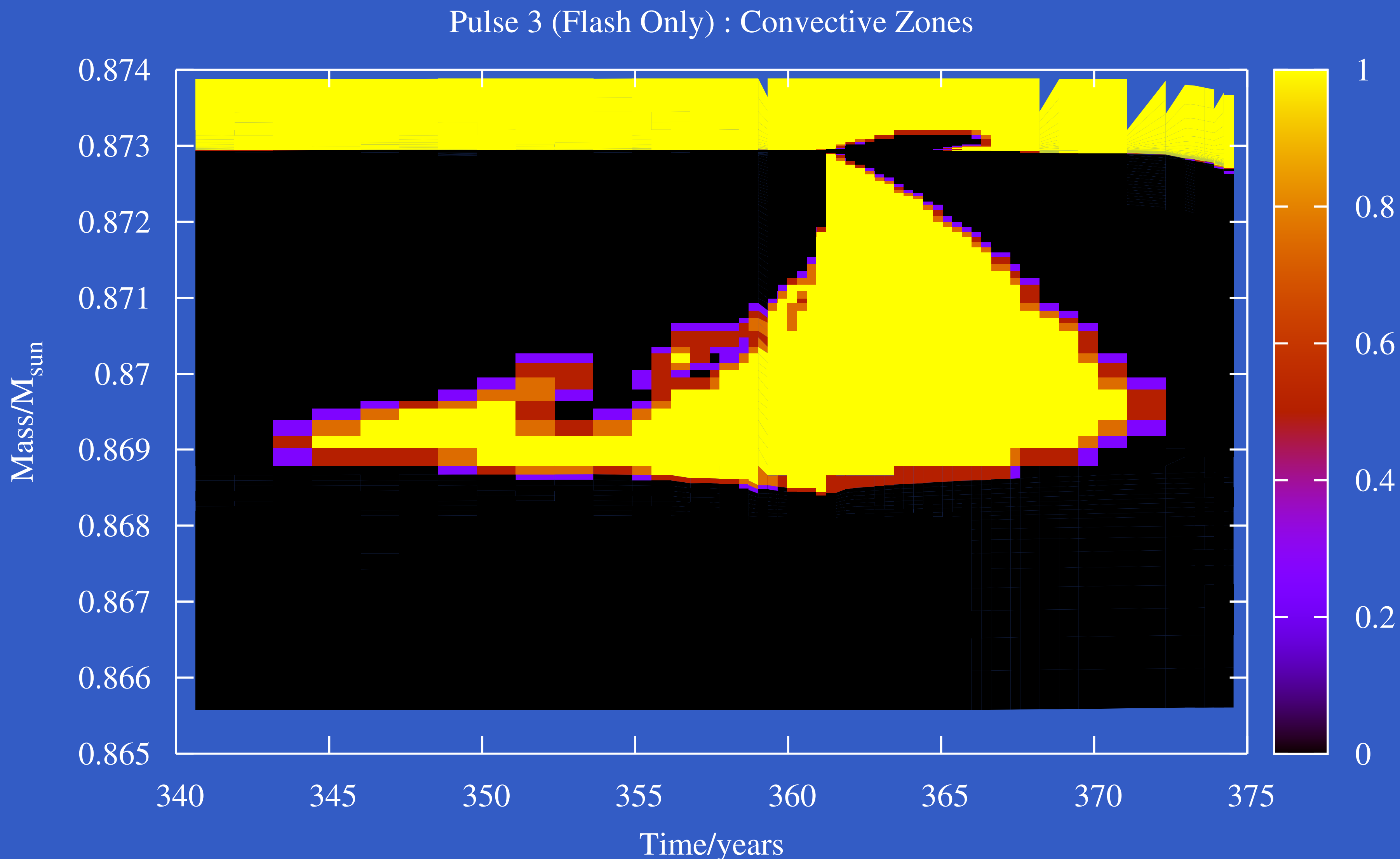
Thermal Pulse 19 T



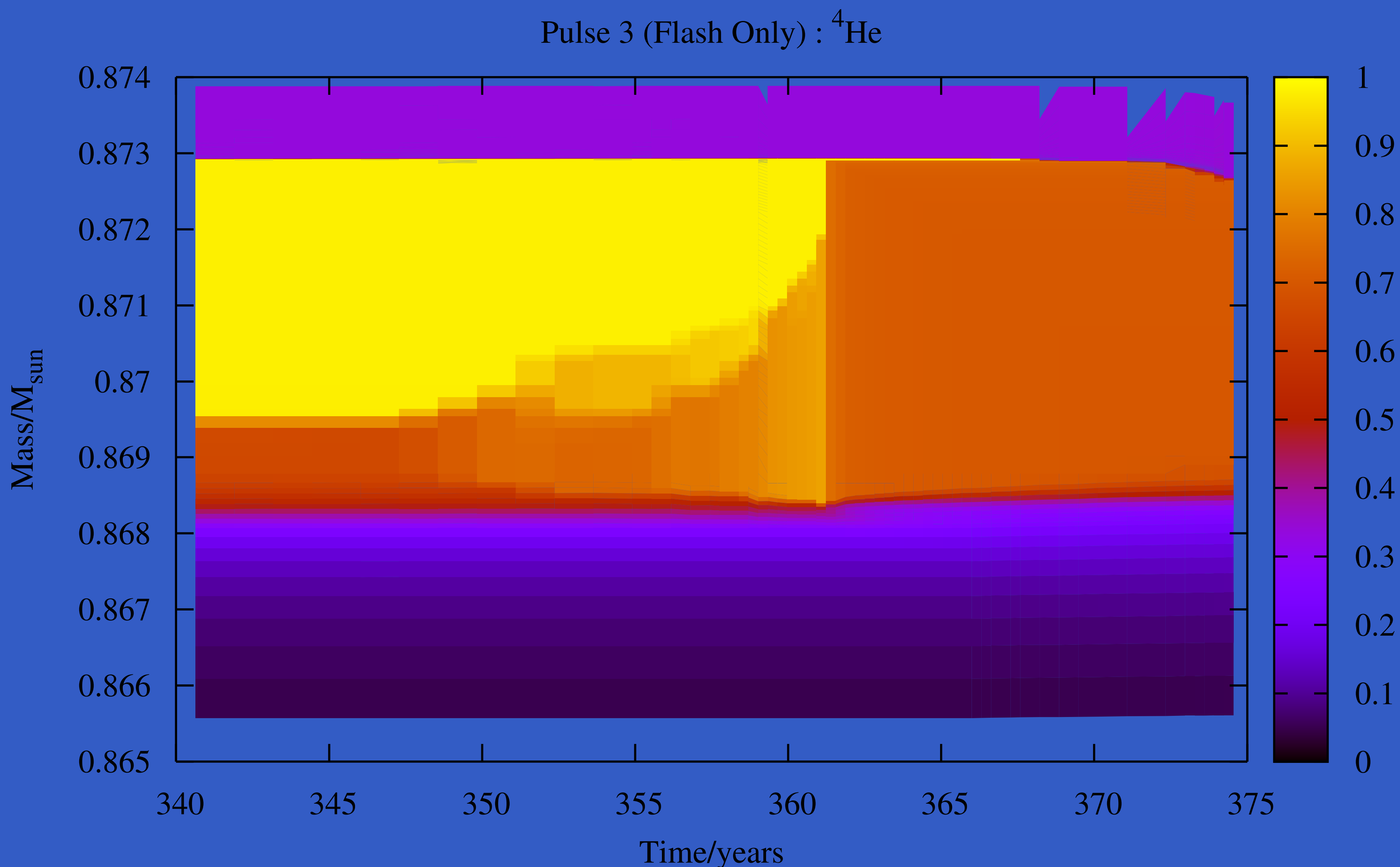
Thermal Pulse 19 ρ



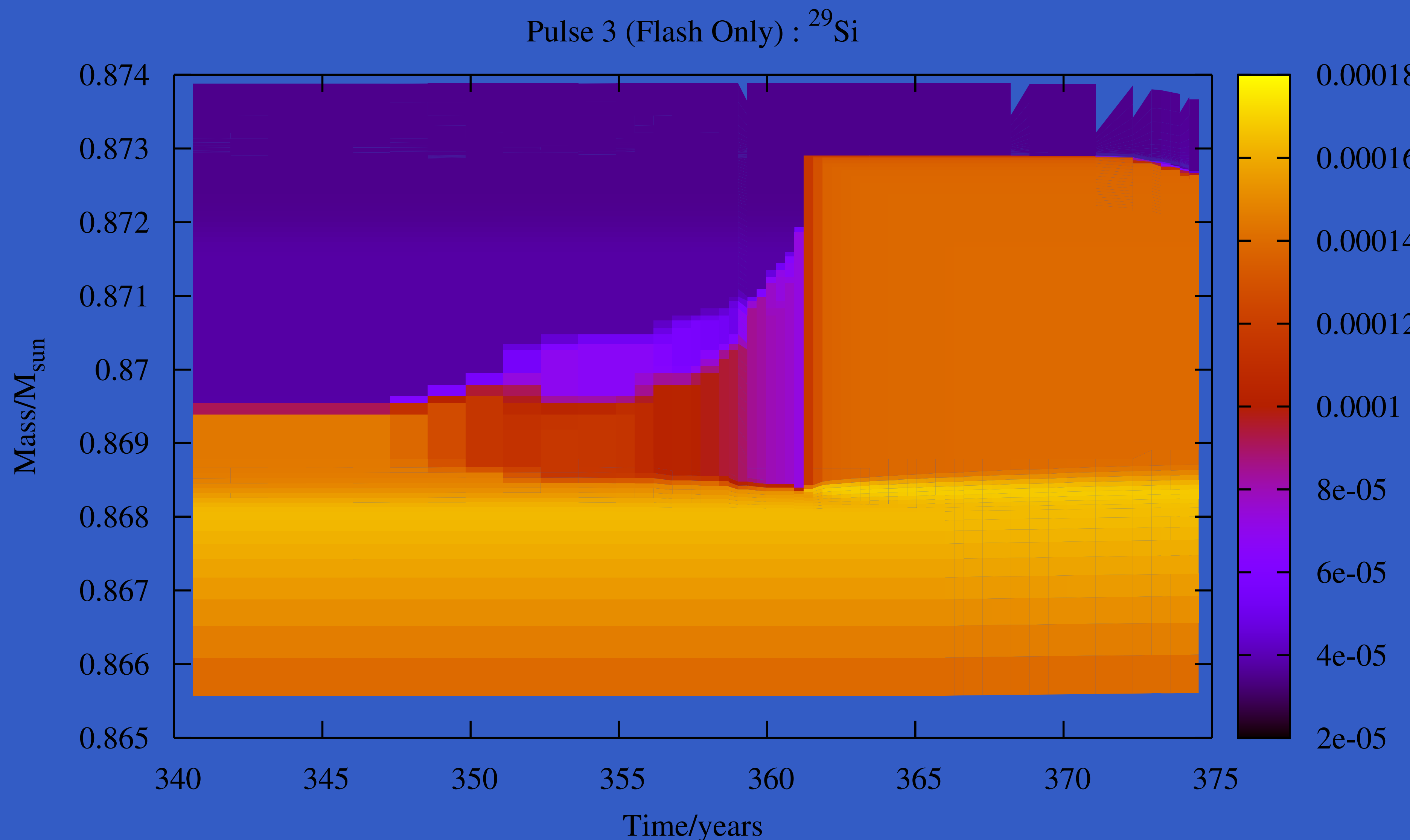
Thermal Pulse 19 Convection



Thermal Pulse 19 ^{4}He



Thermal Pulse 19 ^{29}Si



The end