Stars and Stellar Evolution (WS11-12)
Computer Practicum with WTTS

Virial Theorem Exercise

NOTE: This exercise is a recap of the virial theorem and its importance in stellar evolution. Just to remind it to you the virial theorem states that

$$U + 2T \;=\; 0\,,$$

where T is the kinetic energy of the system and U is the potential energy. In a star, the kinetic energy T is related to the internal energy $E_{\rm int}$ through the adiabatic index γ and gravity is the source of potential energy $U = E_{\rm grav}$, so

$$3(\gamma - 1)E_{\rm int} + E_{\rm grav} \;=\; 0\,,$$

where the adiabatic index $\gamma = 5/3$ for an ideal gas. We want to investigate stellar models to test the virial theorem and the validity of the ideal gas approximation during various stages in the life of a normal star (like our Sun) or more massive stars (e.g. the models used in the previous exercise).

1. Find your solar mass model(**1M**) **in your *working** directory to carry out the following tasks:

1. Use the **HRD** to select an appropriate model for the **main sequence**.
2. Using the **Internals tab**, save the model as `1Msun_MS.mdl`(just by right clicking on the model number you have chosen) files **in your *working directory**.
3. Find the **perl script** "`virial_theorem.pl`" in your ***working** directory. You have to edit the script to make it work with your `*.mdl` input files.
   a) Using your favourite text editor, e.g. *emacs* or *gedit*, change the filename in the perlscript i.e. find the line `$input_file=''*.mdl'';`
   b) Write expressions (in Perl!) to calculate the internal and gravitational energies by using the correct expression for $E_{\rm grav}$ and $E_{\rm int}$ i.e. `$E_grav -=.....;` and `$E_int +=.....;`
4. Comment on your results obtained for $E_{\rm grav}/E_{\rm int}$ (from the model) with regards to validity of the ideal gas approximation.
5. How does the α factor to compare with the result you obtained in the exercise 2.3(a)? [e.g.$\alpha = 1.5$ for the main-sequence (ref. Maeder)]

2. Using the same prescription, select a **main sequence model** for a high mass star (e.g. from last class) to find value of γ and the appropriate equation of state.

3. The total energy of the star is $E_{\rm tot} = E_{\rm int} + E_{\rm grav}$. Derive an expression for $E_{\rm tot}$ as a function of γ and either $E_{\rm int}$ or $E_{\rm grav}$. From this, deduce what happens if the star has $\gamma < 4/3$.