# Stellar Evolution with WTTS

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## 1 Introduction

Welcome to the *Stellar Evolution with Window To The Stars* examples. In the following we guide you through the process of making your own stellar models in a variety of scenarios. You will make models, analyse them, and at the same time learn about how stars work. The software we will use is called *Window To The Stars*. It is a free user interface to a stellar evolution code called *TWIN*. You can find details of the code, installation instructions and a manual at the website `http://www-astro.ulb.ac.be/~izzard/window/`. Here we will give a brief introduction.

### 1.1 Window To The Stars

*Window To The Stars* (*WTTS*) was developed in 2006 by Rob Izzard and Evert Glebbeek to provide an easy way to use a complicated tool - a stellar evolution modelling program. The physics of stellar evolution is difficult enough, without having to worry about computational details. It runs on PCs using Linux, and on Macs with OSX. It uses only free software and is all open source.

You should go to the *WTTS* website and follow the instructions for installation. If you have problems, email the authors using the address provided at the website.

### 1.2 TWIN

*TWIN* is the single and binary stellar evolution code of Peter Eggleton and collaborators. You will need to install *TWIN*, details are on the *WTTS* website.

### 1.3 Running *WTTS*

In each section that follows it is assumed that you are already running *WTTS* according to the instructions given on the website. Usually you just type *wtts*. Note that *WTTS* will store files (many large files!) in the directory from which you run it so please make sure you have sufficient disk space in the current directory. If in doubt, make a new directory and run *WTTS* from there.

### 1.4 Familiarity with WTTS

Please read the section *Using Window To The Stars* in the *WTTS* manual (available from the website) before starting this course. You do not have to remember the details but it would be very useful for you to get an idea of what is going on and how it works before you dive in.

## 2 Main Sequence of The Sun: $M = 1\,\mathrm{M}_\odot$, $Z = 0.02$

The sun is the nearest star to the Earth so we shall start by trying to make a model of it.

### 2.1 Making the model sequence

First we must make a sequence of $1\,\mathrm{M}_\odot$ models starting from the birth of the sun (time $t = 0$), through the present day (about 4 Gyr after birth) and into the future.

1. Start *WTTS*.

2. We wish to start from a *Zero-Age Main Sequence (ZAMS)* model of the Sun. It is thought the Sun formed (its *"Zero Age")* about $4 \times 10^9 \equiv 4\,\mathrm{Gyr}$ ago from some kind of spinning gas cloud. Unfortunately, we cannot model such formation with *WTTS* – indeed nobody really knows how it happened and the best models require huge 3D hydrodynamics models – but the product of the formation mechanism is just a hydrostatic $1\,\mathrm{M}_\odot$ burning ball of gas. *That* we can model with a stellar evolution code.
3. In the *Options* tab, select the button *From ZAMS Library* so we start with a ZAMS model.
4. Select $Z = 0.02$ from the metallicities menu.
5. Select $M_1 = 1$ from the masses menu. It does not matter what $M_2$ is set to.
6. In *Option Control* press *Reset from defaults*. This sets all the options to their default settings, which should be good enough to make a solar model. We will mess about with the options later on.
7. In the sub-tabs (marked *Operation Mode, Initial Conditions…*) find the *Operation Mode* tab. Make sure *INIT_RUN.ISB* is set to 1 (single star operation) and *INIT_RUN.KTW* is set to 1.
8. Set *INIT_RUN.KPT* to 400. This is the number of time-steps that *TWIN* will run for, in our case 400 is enough.
9. Click on the *Evolve* tab.
10. Click on *Clear Log Window* – this clears the log window of any previous data.
11. Click on *Evolve* and wait a few seconds. You will see the status bar change to *Calling Evolve Script* and then, hopefully, to something like *Evolving 27.21Mb ev (pid 4644)::23.08Mb*. This means that TWIN is running! The *23.08Mb* is the amount of memory *TWIN* is using, *pid 4644* is the process ID of the *TWIN* process (called *ev*) and *23.08Mb* is the amount of memory that *WTTS* is using. These are diagnostics, you probably should only pay attention to them if the numbers get very large, or if some red text appears to indicate something has gone wrong.
12. The status bar has two more lines, marked *Star 1* and *Star 2*. In our case we are not using *Star 2* so it is always marked *Pending…* - just ignore it. Star 1 shows the current model number, time, timestep, mass, metallicity, luminosity, effective temperature and radius. This is updated as the star evolves.
13. After a little while the screen will fill with some text. This is the log of what is going on. Most of it is just numbers, but it is useful to see what is happening (once you know what it means!). You do not need to understand this log, but note that if there is no log then something has gone wrong. INSERT SOMETHING ABOUT THE LOG HERE
14. Click on *Update Log Window* – this refreshes the log. Sometimes it goes wrong and does not update properly so this is useful to check that everything is actually there.
15. Click on *Follow Log*. This option forces the log window to follow the bottom of the logfile, which is the newest text. This is the best way to follow the evolution of your star.
16. If you have a fast computer you can go straight to section 2.2. If your computer is slow you should let the model sequence reach the end (it will stop itself) and then move on.
17. If you wish to stop the evolution you can do so at any time by clicking on the *Terminate* button.
18. You will know when the evolution sequence is finished because the status bar will read (in red) *In Star 2: STAR12 no timesteps required*. This cryptic message just means it is done, you can move to section 2.2.

## 2.2 The Hertzsprung-Russell Diagram

The primary tool of stellar evolution is the *Hertzsprung-Russell diagram* (HRD). This is a graph with the *effective temperature* ($T_{\mathrm{eff}}$) on the $x$ axis and *stellar luminosity* on the $y$ axis. (Both are presented after taking the base 10 logarithm.) We shall examine the HRD of the model sequence you just made.

1. Click on the *HRD* tab. In this tab you should see the HRD of your model sequence.
   What is the initial temperature of your model set? What is the final temperature? What is the maximum temperature reached by any model?

2. You can zoom in and out by manually setting the *X Range* and *Y Range* boxes. Try it! To have the ranges set automatically set the boxes to * (asterisk).
   Locate the point in the HRD corresponding to the sun *now*. What is the luminosity? What is the effective temperature?

3. The *Stellar Colours* button activates a feature which tries to colour your HRD according to real stellar colours. You can turn this on or off as you please (and you can adjust the hue and brightness in the *Misc* tab).
   What colour is the sun when you activate *Stellar Colours*?

4. You can label the curve in the HRD with one of many variables. The default is the *Age*, which is just the age of the star (in years). Note the labels along the curve. You can try selecting something else from the *Label With* dropdown menu. If you do not have enough labels, reduce the *Label Spacing*.
   Again locate the sun *now*. Approximately what is the number of the model which corresponds to the present-day sun? What is its radius? What is the thickness (not the depth) of its convective envelope? What is the luminosity of the model with the maximum temperature?

5. You can change the line width with the *Line Width* menu.

6. If you evolve a binary you can look at the other star, or both, with the *Star* menu.

## 2.3 Stellar Structure

While the HRD tells us something about how a star looks to an observer on the Earth, we are still curious about what is happening in the interior. This is the *stellar structure*. In this section we will examine what is going on inside the star as a function of time.

1. Click on the *Structure* tab.

2. You will see some options on the left and an image panel on the right. By selecting some of the options you can display information about various stellar structure variables, such as the central temperature, chemical composition etc. as a function of (usually) time.

3. On the left, the first drop-down menu is the star number. Set this to *Star 1* because are using single star models.

4. The next drop-down menu is the variable you wish to plot on the $x$ axis. Set this to *Age*.

5. There are two sets of three buttons below this which allow you to modify what you plot on the $x$ or $y$ axis. You can either leave the data as it is (*Linear Y Axis*), take the base-10 logarithm (*Log Y Axis*) or plot $10^y$ (*10^Y Axis* - this is the opposite of *Log Y Axis*). The same rules apply to the $x$ axis.

6. Next are the range boxes which allow you to specify the range of the plots. These allow you to zoom in and out.

7. Finally is a long list of variables which you can plot on the $y$-axis. Try scrolling up and down to see how many there are.

8. Click on *log Central Temperature*. This plots the temperature at the centre of the star vs the age of the star.
   How hot is the centre of the star at age zero? What temperature does the star reach in the final model? What is the present-day central temperature in the sun?

9. The temperature rises until it reaches a plateau.
   At what age is the plateau reached? Approximately what is the central temperature at this time? Make an estimate for the average central temperature $T_c$ over the lifetime of your models.

10. Click *log Central Temperature* to deselect it, and instead choose *log Central Density*.
    Is there a plateau in central density? What is the present-day central density of the sun? Make an estimate for the average central density $\rho_c$ over the lifetime of your models.

11. Deselect *log Central Temperature*, now choose *Central abundance of H*. This shows you the abundance of hydrogen at the centre of the star by mass fraction. This means that if the abundance is 0.5, 50% of the mass is in hydrogen.
    What is the initial abundance of hydrogen? Why is the star not 100% hydrogen at the beginning of its life? What is the rest of the star made of at the beginning? Why does the hydrogen abundance drop with time? At what time is the central hydrogen abundance almost zero? Does this time look familiar to you?

12. Click on *Central abundance of He* but do *not* deselect the hydrogen abundance.
    What is the initial helium abundance? When and where was most of the helium made? Why does the helium abundance increase as a function of time? What is the final helium abundance? What is H+He at $t = 0, 2, 4, 6, 8, 10\,\mathrm{Gyr}$?

13. Deselect the hydrogen and helium abundances, instead select the central abundances of C, N and O (carbon-12, nitrogen-14 and oxygen-16) . You can see N and O but C is very hard to see, so click on *Log Y Axis*. Now that we have taken logarithms of the numbers we can see the changes much more easily.
    What are the initial abundances of C, N and O? What is the sum of the initial abundances, i.e. C+N+O? Why does the C abundance drop so quickly at early times? What does C get turned into and which burning cycle is involved? What is the product of this burning cycle? What happens to the oxygen abundance after 6 Gyr? What does the oxygen get turned into and which burning cycle is involved? What is the product of this burning cycle? What is the sum C+N+O at $t = 0, 2, 4, 6, 8, 10\,\mathrm{Gyr}$? Why is this (almost) constant? Why is it *not* quite constant?

14. The abundance *by number of particles* (i.e. the number density) $N$ is related to the abundance by mass fraction $X$ by $N = X\rho/m$ where $\rho$ is the density and $m$ is the nuclear mass of the species (in this case, $^{12}\mathrm{C}$, $^{14}\mathrm{N}$ and $^{16}\mathrm{O}$).
    Given the density you estimated for the centre of the sun and the initial abundances of C, N and O, what are the initial number densities $N_{\mathrm{C}}$, $N_{\mathrm{N}}$ and $N_{\mathrm{O}}$ in units of $\mathrm{cm}^{-3}$?

15. We can deduce why you see the abundance changes. The reactions $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$ and $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ destroy $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ as you have seen in the models. The rate equations are (approximately)

$$\frac{dN_{\mathrm{c}}}{dt} = \frac{-N_{\mathrm{c}}}{\tau_{\mathrm{C}}}, \tag{1}$$

$$\frac{dN_{\mathrm{O}}}{dt} = -\frac{N_{\mathrm{O}}}{\tau_{\mathrm{O}}}, \tag{2}$$

    where $\tau_{\mathrm{C}}$ and $\tau_{\mathrm{O}}$ are timescales for the reactions. These equations are similar to radioactive decay equations and you should be familiar with the idea of radioactive decay timescales and the half-life. The timescales are given by $\tau = 1/(N_{\mathrm{H}}\langle\sigma v\rangle)$ where $N_{\mathrm{H}}$ is the number density of hydrogen and $\langle\sigma v\rangle$ is the reaction rate cross section.
    What is the approximate number density of hydrogen, $N_{\mathrm{H}}$, in units of $\mathrm{cm}^{-3}$, at the beginning of the evolution?

16. At temperatures around $T = 0.016 \times 10^{9}\,\mathrm{K}$, the values for $N_{\mathrm{A}}\langle\sigma v\rangle$ are $8.96 \times 10^{-16}$ and $3.65 \times 10^{-20}$, for $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$ and $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ respectively, where $N_{\mathrm{A}} = 6 \times 10^{23}\,\mathrm{mol}^{-1}$ is the Avogadro number.
    Given that $\tau$ is in seconds, what are the units of $\langle\sigma v\rangle$? Calculate $\langle\sigma v\rangle$ for each reaction. Calculate $\tau_{\mathrm{C}}$ and $\tau_{\mathrm{O}}$.

17. We can calculate the rate of destruction of $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ from equations 1 and 2.
    Calculate the rates $dN_{\mathrm{C}}/dt$ and $dN_{\mathrm{O}}/dt$. Estimate the timescale of destruction of carbon and oxygen from $\frac{N_{\mathrm{C,initial}}}{dN_{\mathrm{C}}/dt}$ and $\frac{N_{\mathrm{O,initial}}}{dN_{\mathrm{O}}/dt}$ in both years and in Gyr. Does this explain what you see in the C, N and O abundance vs time plot? Why are your estimates likely to be wrong?

18. You can use a different $x$ coordinate from *Age*, for example *Model Number* (which increases non-linearly with the stellar age). *WTTS* is completely flexible in this sense.

19. Sometimes this leads to strange results, e.g. curves which are not monotonic.

20. A useful diagnostic is the *log central Temperature* vs *log Central Density* plot. Select this plot (with a *Linear Y axis*).
    Which way does the curve evolve with time? Why is it initially linear? What happens when the temperature reaches $\log T \sim 7.27$? Have you seen this before?

21. Experiment, make your own plots, can you explain everything you see?

## 2.4 Internal structure of the star

In the previous section we looked at the evolution of stellar structure from the point of view of a few special variables (e.g. central or surface temperature/density etc.). However, you will often want to know what is going on at any point in the star. In the *Internals* tab you can do this in a number of ways.

1. Click on *Internals*.
2. You will see a new tab window with some labels across the top, options on the left and a plot on the right.
3. The buttons across the top are the *Y Axis* (labelled $M$, $R$, $P$ etc.). Hover your mouse pointer over each button to get a description of what it means. By selecting one or more of these you can choose what to plot on the $y$-axis.
4. On the left are many options, we will try a few. First, make sure *Star 1* is selected. This is the familiar selector you have seen before, but of course we still only have one stellar model.
5. You can make animations or still images. Click on *Still*.
6. There are several ways to visualise data, some are better than others. Click on *Line*.
7. Ignore *Frame* and *Speed* for now, they are controls for when we animate (later!).
8. Next you come to two drop-down menus. The first selects how we want to identify models, leave this at *Model Number*.
9. Second is the Abscissa ($x$ axis) coordinate selector. Leave this at $M$ (the mass coordinate) although of course you can change it to any coordinate you like (with strange results if the coordinate is not single valued!).
10. Next are the range boxes for the $x$ and $y$ axes. You can also click on the *Log 10 . . .* buttons to take base 10 logarithms.
11. Now we come to the useful part. In order to see inside the star we have to look at a stellar model, which is really a snapshot of the stellar history. In our case we made 400 models, which are labelled from 2 to 401. You can select which of these models you would like to plot from the scroll box which lists them. You can select more than one. If you click on a model, hold shift, and click on another you can select a range. Try it.
12. The buttons marked *C*, *1*, *2*, etc. allow you to select models automatically. *C* clears the list, *1* selects them all, *2* selects every 2 models, and so on. The vertical bar | (which means *or*) inverts the selection. Experiment with these.
13. The *Latest* model always points to the final model in the evolution run. In our case this is number 401.
14. Press *C* to clear the list. Select *Latest* and model number 2 from the list (these are the first two models).
15. Because you have not selected anything from the *Y axis* buttons, you will see nothing (*No Image* will be in the display). So, go ahead and press the *T* (temperature) button from the *Y Axis* buttons at the top. Make sure the Abscissa menu is set to *Abscissa: M*.
16. You now see two curves, one is the temperature as a function of mass for model 2 and one is for model 401. You can see that the star has become hotter at the centre and cooler at the surface.
17. Press *C* to clear the list, and now press *10* to select every 10th model. The graph shows 40 curves simultaneously, which is not at all easy to understand! (Set the lower *Ordinate Range* to 6, this will help.)
18. For this reason you can animate the curves. Press *Animate*. Set *Speed* to 11 to make the animation as fast as possible. *WTTS* will draw each of the 40 curves in its own window – this takes some time – and as it does so you should see the *Frame* number increase. Eventually all the curves will be drawn and the animation will loop. Try changing the *Speed* – a value of 11 is as fast as your machine can do it (you can make it faster by using PNG instead of Postscript images, which you can select in the *Misc* tab).

19. Set the *Speed* to 0 and try manually scrolling through the different frames with the *Frame* button.
20. Ignore the *Sphere* and *Slice* options – these did once work but are currently buggy.
21. Finally, there are 20 buttons above the plot. Normally you work in plot 0 but you can actually have 20 simultaneous plots. Try choosing a different number, make a new plot and then try again.
22. Now press *C* to clear the list, set the *Ordinate range* to automatic (put an asterisk * in both boxes) and de-select *T* in the *Y Axis* buttons.

We will now look at the sun *as it is today*.

1. In section 2.2 you noted the model number of the present day sun. Find this model in the list and select it.
2. Plot luminosity $L$ vs mass coordinate $M$. Select the *Ordinate range* to be 0 to 1.1.
   What is the luminosity at the centre? What is the luminosity at the surface? What should it be? What is the luminosity at $M = 0.5$?
3. Plot luminosity $L$ vs radius $R$ (select *Abscissa: R*)
   What is the radius at the centre? What is the radius at the surface? What should it be?
4. Deselect $R$, choose *Abscissa M*, reset the *Ordinate range* (to * and *), select *Log 10 Ordinate* and press the *E_th*, *E_nuc* and *E_nu* buttons.
   What is *E_nuc*? Why is it concentrated in the centre? (Hint: try changing the *Abscissa* to *T* or *rho*).
   What are *E_th* and *E_nu*? In the grand scheme of things, are they important in the Sun? Deselect *Log 10 Ordinate* to check your conclusion. How do we observe *E_nu* on the Earth? What theory had to be modified because of *E_nu*?
5. Deselect everything. Plot *Grad_rad-Grad_ad* ($\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}$) as a function of $M$ – this measures the convection in the star (use the *log* buttons if you need to). Where it is positive there is convection, such as near the surface.
   Observations of pulsations in the sun lead to the claim that the outer 30% of the sun is convective. Does this plot support this view? If not, what do you think is meant by the "outer 30%" and can you make a plot to confirm this? Is the number 30% correct, if not can you suggest a reason why not?
6. Change the abscissa from $M$ to *opacity*. You probably need to log both axes.
   What is the relationship between $\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}$ and opacity? Is this due to changes in $\nabla_{\mathrm{rad}}$ or $\nabla_{\mathrm{ad}}$, if so why do these changes occur? At what temperature is the opacity the greatest? Where is this in the star (find the $M$ and $R$ coordinates)?
7. Finally, clear the plot and select $S$ as a function of $M$.
   What is $S$? Why is it low in the centre? Has it increased or decreased in the star from the beginning of the evolution to now? What about the rest of the universe?

## 2.5 Kippenhahn Diagrams

The final *WTTS* tab we are going to look at is the *Kippenhahn* tab. A Kippenhahn diagram is a plot of time (or model number) on the $x$ axis, mass coordinate on the $y$ axis and a colour or shading to indicate convective regions in the rest of the plot. In *WTTS* this idea is extended to allow *any* variables on the $x$ and $y$ axes, and *any* variable for the colour (mapped surface) plot.

First we will make a traditional Kippenhahn, then try making some fun colour plots.

1. Select the *Kippenhahn* tab.
2. You should see the usual *Star 1/2* selector, just leave this at *Star 1*.
3. You then have the $x$, $y$, $z$ axis settings ($z$ is the colour surface which will be plotted, equivalent to the convective regions in the canonical Kippenhahn diagram).
4. You can choose the variables, and whether to plot them in a linear, log or $10^x$ fashion, from the drop down menus. The variables which are available to you are the same as those in the *Internals* tab, so it is assumed you have been through that section.

5. The ranges can also be set in the boxes (autoscaling is again marked with an asterisk *).
6. For the $x$ and $y$ axes you can set the *resolution*. A setting of 100% means that every point is plotted, a settings of 10% means that every tenth point is plotted. A lower resolution will plot more quickly, so is useful for a quick sketch. Setting *WTTS* to high resolution may take a long time to plot, especially for long model sequences, because the amount of data which must be accessed is very large. This effect will be even worse if you are using a slow hard disk or your data is being transferred across a network (you have been warned!).
7. The *Palette* section allows you to change the colours.
8. Next are the *Show...* buttons. These are *only* useful if your $y$ coordinate is the mass $M$. They allow you to plot mass boundaries (the surface, core etc.), convective boundaries and nuclear burning zones. These are really for the expert user, but perhaps you will find them useful.
9. The *Replot* button is where the action happens. Unlike all the other plots in *WTTS*, the Kippenhahn diagram does *not* plot itself when you change something. The reasoning behind this is that the replot may take a very long time, so if a continuous replot was to happen it would slow you and your machine to a crawl. You can also replot by pressing the *r* key.

### 2.5.1 Traditional Kippenhahn Plots

1. Select *Age* for the $x$ axis, $M$ for the $y$ axis and *Convection* for the $z$ axis. Select *Log10* for the $z$ axis. The special variable *Convection* is actually max $\left[\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}, 10^{-30}\right]$, so is positive when there is convection, and tiny when there is not. Taking the log means we show only the convective regions. Set the resolutions at 10% and 10%. Hit replot.
2. You will see mostly black, which corresponds to $-30$ in the colour key. This is because $\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}$ is negative in these regions and the logarithm of a negative number is not possible (in this context at least!). To cope with this *WTTS* sets $\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}$ to something very small (in this case $10^{-30}$ which logs to $-30$). You can get around the problem by setting the $z$ range minimum to 0. Hit *replot*.
3. Now you see a coloured band across the top, but it is very jerky. Set both the resolutions to 50% (and hit *replot*) and there will be fewer jerks. Try 100% – this is the best we can do.
4. Remove the range setting, set the resolution to 100% and press *replot.*
   What is the depth of the convective envelope in the Sun? Does it vary over most of its lifetime?

### 2.5.2 Enhanced Kippenhahn Plots

1. We can compare to the work of the previous sections. Set the $y$ axis to plot the radius $R$ instead of $M$. Hit *replot*.
2. Now you can see that the *depth* of the envelope is nearer the 30% often quoted – it depends on whether we use $R$ or $M$ as a coordinate.
3. You can of course plot anything in these diagrams. Try plotting model number on the $x$ axis, $M$ on the $y$ axis and $T$ on the $z$ axis. You can clearly see the temperature increase in the centre towards the end of the model run.
4. Plot the same thing with $R$ as the $y$ coordinate.
5. Replot with $M$ as the $y$ coordinate, and the oxygen abundance as the $z$ coordinate.
6. Plot *Age*, *M* and *E_nuc* (all *Linear*). This shows the nuclear burning regions.
   Nuclear burning starts in the core. What happens to the burning region when the core runs out of hydrogen? Hint: try setting the $y$ range maximum to (say) 0.3to focus on the central region.
7. Change *Age* to *Model Number*. Replot. You can see that it is much easier to see the details of the transition from core to shell burning when plotting against *Model Number*. This is often the case and is a trick worth remembering.
   Why is the model number not simply linearly proportional to time?
8. Experiment with the palette. Some colours will be better than others, it depends on what you want to see.

# 3 The ZAMS

Now that you know how to use *WTTS* we are going to move from a $1\,\mathrm{M}_\odot$ model to a complete *Zero Age Main Sequence* model set. This is a set of models which are all at the beginning of their evolution (*Zero Age*) but vary in mass. In this case we are going to make models between 0.5 and $20\,\mathrm{M}_\odot$.

## 3.1 Constructing the ZAMS models

1. Start *WTTS*
2. Click *Reset from defaults*
3. In the *Options* tab, find the *Mass Loss and Gain* subtab. Set *INIT_DAT.CMI* to 1e-9 (this means $10^{-9}$ in floating point notation).
   *CMI* is an artificial mass gain rate in $\mathrm{M}_\odot\,\mathrm{yr}^{-1}$. Why do we want to set this to be non-zero? What is the qualitative effect of adding mass onto a main sequence star? (Other than just increasing its mass…)
4. In the *Mass Loss and Gain* tab set *INIT_DAT.CMR* and *INIT_DAT.CMJ* to 0. This turns off mass loss.
   Why do we do this?
5. In the *Operation Mode* tab set *INIT_RUN.KPT* to 1095.
6. Find the *Artificial Physics* tab and set *INIT_DAT.KTH* to zero.
   *KTH* multiplies the thermal energy term in the stellar structure equations. Why should we set this to zero?
7. Find the *Nuclear Network* subtab (you will have to scroll to the right using the arrow). Set *INIT_DAT.KCN* to 1.
8. Select the nearest initial mass to $0.1\,\mathrm{M}_\odot$ (also make sure *From ZAMS Library* is set, and that $Z = 0.02$).
9. Go to the *Evolve* tab. Hit *Evolve*.
10. Click on *Follow Log* to see what's going on (you may need to click *Update Log Window* to get it going). You can either follow the log (if you can work out what it means) or just look at the status bar. Because we are evolving *Zero Age* models the age given by $t = \dots$ is spurious and does not conform to any real time. You should instead look at $M = \dots$ which gives the current mass, which should be going up.

## 3.2 HRD of the ZAMS

1. Go to the *HRD* tab and construct an HR diagram of the zero-age main sequence which is labelled by the mass. Play with the line width and the label spacing to make the HRD look as good as possible.
2. Questions: What colour are low mass main sequence stars? What colour are high mass main sequence stars?
3. Print your HRD and label the spectral types O, B, A, F, G, K and M.
4. What was the temperature and spectral type of the Sun when it was born?
5. Estimate a functional form for $L(T_\mathrm{eff})$ for the ZAMS. Use this to estimate a function $R(L)$ where $R$ is the stellar radius.

## 3.3 Mass-Luminosity relation

1. Go to the *Structure* tab.
2. Plot *Mass* on the $x$-axis, *log Luminosity* on the $y$-axis.
3. Questions: Estimate an expression for the luminosity $L$ as a function of mass $M$ in the form $L \propto M^n$. Estimate how much hydrogen there is in a star of mass $M$. Based on this, estimate how much energy $E$ is available for nuclear burning of hydrogen as a function of $M$. Estimate the timescale for nuclear burning $t \propto M^p$ (assume that $L$ is constant).
4. Instead of luminosity, plot radius. Estimate a function $R(M)$. Also estimate a function $R(L)$ - is there a correlation? Do you expect one? Does this match your previous estimate?

### 3.4 Convection

1. Go to the *Kippenhahn* tab.
2. For the $x$-axis choose *Mass* and make sure it is logged.
3. For the $y$-axis choose *M/Mass* (linear).
4. For the $z$-axis choose *Convection* and make sure it is logged.
5. Hit *Replot*.
6. Questions: What is the minimum mass for a star to have a convective core on the ZAMS? What is the maximum mass for a star to have a surface convection region?
7. Try plotting $T$ instead of convection (hint: try setting the $z$-Range minimum to 6 to get better contrast). Why are higher mass stars hotter?
8. Plot *rho* (the density) instead of $T$ (remember to reset the $z$-Range! Try setting the minimum to $-2$ to get better contrast). Do you expect what you see?
9. Plot *opacity* instead of density.
   In the *low mass* stars (try zooming in on the $x$ axis) why does the opacity get so high near the surface?
   In high mass stars, is the opacity the reason for the existence of a convective core?
10. Plot *E_nuc* (you should set the ranges to suitable values because some regions of the star will have *E_nuc* set to a very small number, effectively zero). Does this plot remind you of a previous plot? What is the reason for convection in the cores of high mass stars?
11. You can split *E_nuc* into the contributions from the *pp* and CNO burning cycles. Instead of plotting *E_nuc*, try plotting RPP (the *pp* chain burning rate), RPC (the $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}(\beta^+,\nu)^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ burning rate), RPNG (the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}(\beta^+,\nu)^{15}\mathrm{N}(p,\gamma)^{16}\mathrm{O}$ rate), RPN (the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}(\beta^+,\nu)^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}$ rate) and RPO (the $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}(\beta^+,\nu)^{17}\mathrm{O}(p,\alpha)^{14}\mathrm{N}$ burning rate). What is the relative contribution to the nuclear burning rate from each reaction as a function of mass?
12. For both the *pp* chain and CNO cycle the reaction rate is approximately proportional to a high power of the temperature. What are the powers for each reaction cycle? Check your $T$ plot - can you explain the convection in higher mass cores now?

### 3.5 Temperature and Density

1. Go back to the *Structure* tab.
2. Plot *log Central Temperature* on the $x$ axis and *log Central Density* on the $y$ axis (both linear plots).
3. Why does the central density drop for $\log_{10} T_\mathrm{c}/\mathrm{K} \gtrsim 7.3$?
4. If you cannot work this out, try plotting *Mass* (logged) on the $x$ axis. At what mass does the density start to drop? Have you seen this mass before? What is the structural difference between stars above and below this mass?

## 4 Post-MS Evolution in the $1\,\mathrm{M}_\odot$Star

### 4.1 The End of the Main Sequence

1. Evolve a star of $1\,\mathrm{M}_\odot$ (and $Z = 0.02$) as far as it can go. You should be able to use the default settings, select $M = 1\,\mathrm{M}_\odot$ from the ZAMS library and just evolve it.
2. Make an HR diagram. Label it with the *Age* of the star. Print it out so you can work on it and label points by hand.
   Does a $1\,\mathrm{M}_\odot$ star get redder or bluer as it evolves from the ZAMS?

3. The *Main Sequence* ends when the star runs out of hydrogen in the core, although the definition is not precise. Let us say this is when the central hydrogen abundance is $< 10\%$. Set the labels on your HRD to *Central H abundance* – find the approximate location of the 10% point and label it on your printed HRD. How long does a $1\,M_\odot$ star spend on the main sequence? Approximately what fraction of its total lifetime is this?

4. At the end of the MS the star reverses its change in temperature which reflects an internal structural change. Go to the *Structure* tab and plot *log Central Density* vs *Age*.
   What happens to the central density of the star at the end of the MS? Why does this happen? (Hint: plot *Central abundance of H*)

5. How much does the central temperature change during the main sequence? If there is a change, why?

## 4.2 Hertzsprung Gap

The Hertzsprung Gap is the transition from the Main Sequence to the Giant Branch.

1. Go to the *Kippenhahn* tab.

2. On the $x$ axis plot the *Model Number* (linear) and set the range from 0 to 480.

3. On the $y$ axis plot $M$ (the mass coordinate, linear) and set the range from * to *.

4. On the $z$ axis plot *E_nuc* (the nuclear burning, log it) and set the range from -10 to *.

5. Hit *Replot*.

6. From models 1 to 200 would you describe the nuclear burning as *central* or *shell* burning?

7. After model 200, this changes. By model 500 would you describe the nuclear burning as *central* or *shell* burning?

8. After the main sequence, the star moves across the *Hertzsprung Gap* to the red side of the HR diagram. Estimate the time it takes to get from the left ($\log T_{\text{eff}} \sim 3.76$) to the right ($\log T_{\text{eff}} \sim 3.7$) side of the HRD based on your HR diagram. Imagine now that you observe a large number of $1\,M_\odot$ stars of random ages (with a very good telescope and perfect seeing). How many will be in the Hertzsprung Gap? Why do you think it has this name?

9. The Hertzsprung Gap coincides with a structural change in the star due to exhaustion of the core hydrogen. From the *HRD* tab find the model numbers which correspond to the beginning and end of the Hertzsprung Gap (these should be approximately 280 to 480 but check!). Go to the Internals tab and plot the density (*rho*) for some models (every 10 is good, so 280, 290, 300…480) in this range. Set the log scale on the $y$ axis (Ordinate) and set the $y$ range from 0 to 6. Click on *Animate* (and set the speed to something comfortable).
   Questions: By what approximately factor does the central density increase during the Hertzsprung Gap? What happens to the density outside the stellar core during the Hertzsprung Gap? What does this imply for the radius of the star? You can check by plotting *log Radius* vs *Age* in the *Structure* tab.

## 4.3 Giant Branch

During the Giant Branch the star is burning hydrogen in a shell rather than in the core.

1. Plot the same Kippenhahn diagram as you did for the Hertzsprung Gap but set the $x$ range from 400 to * and the $z$ range from 0 to *. You should clearly see the burning shell formation between models 400 and 900.
   What isotope is burning in the shell? What is it being converted into? Which burning cycle is most active? Estimate the thickness of the burning shell around model 1000.
   What is the difference in temperature, density and entropy between the lower and upper sides of the shell at model 1000?
   What are the abundances of hydrogen, helium, carbon, nitrogen and oxygen above and below the shell? Can you explain the changes?

2. In the *Structure* tab, plot the radius $R$ vs time.
   Given that the core is contracting, can you give a reason why the envelope is expanding?
   At what age, model number and stellar mass will the radius exceed one astronomical unit? (And what are the implications!)
3. In the *Structure* tab, plot the mass vs time. What happens to the mass of the star at the end of the Giant Branch? Can you qualitatively explain this?
4. Plot the surface abundances of H, He, C, N and O as a function of model number. What happens around model 700? Hint: try plotting a Kippenhahn diagram for models 600 to 1200, note the mass coordinate of the base of the convective envelope. Then switch from plotting convection to plotting nitrogen. Can you explain this?
5. Estimate the lifetime of the star on the giant branch and compare it to the Hertzsprung Gap lifetime.
6. Why should giant branch stars be easily visible? Should you see more Giant Branch stars or Hertzsprung Gap stars in a random survey?
7. Make a plot of $\log M_c$ vs $\log L$ and derive an approximate function $L \propto M_c^x$.

### 4.4 Central Temperature and Density

It is possible to understand much of stellar evolution from a central temperature vs density plot.

1. In the *Structure* tab plot central temperature vs central density.
2. Can you identify the direction of evolution?
3. Can you label the main sequence, Hertzsprung gap and giant branch phases?
4. At what density does material become degenerate?
5. What is the temperature at the end of the evolution? Is this the hottest part of the star? Try plotting *log Central Temperature* and *log Maximum Temperature* vs *model number*.
6. Plot *luminosity of hydrogen burning* and *luminosity of helium burning* (on the same plot) vs *model number*. Why is the *helium* luminosity going up at the end of the evolution?
7. Go to the *Internals* tab. Select the *Latest* model from the list, which in this case is the final evolution model. Plot the temperature vs mass. Does it surprise you that the hottest part of the star is *not* the centre?
8. The temperature is rising in a region off-centre due to the helium burning: what will be the response of the material around the helium ignition region? Compare a plot of (log) *Central Pressure* vs *log Central Temperature.*
9. In the core the density is high enough that the material is (partially?) degenerate. Can the helium burning be stopped?
10. In our $1\,\mathrm{M}_\odot$ models a numerical instability stops the evolution – speculate on what will happen to a *real* $1\,\mathrm{M}_\odot$ star after helium ignition.

## 5 Case Study: $M = 5\,\mathrm{M}_\odot$

Make a model sequence of a $5\,\mathrm{M}_\odot$ star, at solar metallicity, starting at the zero-age main sequence and going as far as you can.

### 5.1 HR Diagram

Plot the HR Diagram. Can you label the different evolutionary phases? (Main Sequence, Hertzsprung Gap, Giant Branch, Core Helium Burning, Horizontal Branch, Early Asymptotic Giant Branch, Thermally Pulsing Asymptotic Giant Branch, White Dwarf Track). For those which you can identify, use the HRD to calculate the lifetime of each phase of evolution. Compare to your $1\,\mathrm{M}_\odot$ model and try to justify any differences.

### 5.2 Core masses and burning Shells

Plot the helium core mass as a function of time. What nuclear reactions increase the mass of the helium core? What is the helium core mass at the base of the giant branch? What is the maximum helium core mass and when does occur?

Plot the CO core mass as well as the helium core mass, but against model number instead of time. What nuclear reactions make the carbon and oxygen? What happens to the other elements? Does the evolution of the CO core look similar to the He core? What are the differences, and why do they happen? Compare the core temperature during helium burning compared to during hydrogen burning. Which is hotter and why? Compare the maximum to central temperature – when in the evolution, where in the star and why are they different? Why would the centre of the star be cooler than the hottest point?

Use the Kippenhahn tab to plot nuclear energy generation (*E_nuc*) as a function of mass and model number. Identify the model ranges which show a *core burning* and a *shell burning* structure. Compare to your HRD. What are the primary characteristics of shell burning phases? What is peculiar about the *final* burning stage? Do you think the models you have made match real stars?

Make a plot of the abundances in the CO core at the end of the evolution (Hint: use the *Internals* tab). Approximately what percentrage of the core is carbon, and what percentage is oxygen?

### 5.3 Convection

Identify regions of convection in the star as a function of mass and model number. Make a plot to illustrate what you find. What is the cause of the convection? Identify the regions of the HRD which correspond to models with a convective envelope.

### 5.4 Nucleosynthesis

Plot the surface abundances of H, He, C, N and O vs model number. Describe the changes you see, try to identify the reason for the abundance changes. Is there a correlation between your previous findings and the surface abundance changes? Find the model numbers either side of the first major change in surface abundance, and in the *Internals* tab plot the abundance of C, N and O as a function of $M$ (mass coordinate) for a selection of models. Animate the surface abundances so you can see what happens as a function of time. What process is responsible for the initially curved abundance profiles? What process is responsible for the flattening of the abundance profiles? What is conserved in the flat regions? Why does carbon increase in the core after the surface abundances stop changing?

Nucleosynthesis in stars is responsible for the creation of the elements heavier than lithium. What would you expect this $5\,\mathrm{M}_\odot$ star to make? Estimate the *mass ejected* as each of the elements: H, He, C, N, O, Ne and Mg. Compare these to the solar system abundances and terrestrial abundances. Do they match?

Compare the final abundances to the initial abundances and estimate the *production yield* of the elements H, He, C, N, O, Ne and Mg in $\mathrm{M}_\odot$. What does a $5\,\mathrm{M}_\odot$ star make? What does it destroy? Given that it destroys some elements which are very common on the Earth, and that a similar destruction process occurs in most stars, how do you think these elements are made?

### 5.5 End of Evolution

The evolution sequence ends when the stellar evolution code can no longer find a solution to the equations of stellar structure. What is the mass of the star at this point? Why has it decreased? What happens to a real star at this point?

### 5.6 The fate of this star

What is the final fate of intermediate-mass stars? (Hint: What did you find in section 5.5?) In section 5.2 you found that after hydrogen was exhausted in the core and there had been some shell hydrogen burning, helium ignited. Similarly, after helium is ignited in the core it goes on to burn in a shell. What would you expect to ignite after helium? Why does this *not* happen in intermediate mass stars?

Use the *internals* tab to find the average density of the CO core in the final model. Use this to calculate the *binding energy* of the star.

The nuclear masses of $^{12}$C, $^{16}$O and $^{56}$Fe are $2.01 \times 10^{-23}$ g, $2.66 \times 10^{-23}$ g and $9.29 \times 10^{-23}$ g respectively. Given the abundances in the core (which you found in section 5.2) estimate the amount of energy which would be released if the CO core is completely burned to iron. If this happens, will the star remain bound? If not, what would it look like? Estimate how much iron would need to be produced to disrupt the core.

In a normal star, with a *hot* core, what would happen if the carbon ignites? What would happen in a *cold* core if carbon ignites? How would you make a cold CO core? How would you then ignite the carbon? If you were to observe such an ignition, what would you expect to see? What is the common name for this? Why is this process relevant to cosmology and the question of whether universal expansion is accelerating or not?

## 6 Massive Stars

Massive stars do not end their lives after helium burning because their cores are hot enough to ignite carbon and then neon, oxygen and silicon. The result is an iron core which cannot burn exothermically. Our stellar evolution code is somewhat limited and cannot follow the star beyond some of the carbon burning, but we can still investigate the properties of massive stars.

### 6.1 A $10\,\mathrm{M}_\odot$ star

Evolve a $Z = 0.02$, $M = 10\,\mathrm{M}_\odot$ star from the ZAMS as far as it will go. Does this star ignite helium? Does it ignite carbon? Make a Kippenhahn plot of *E_nuc* to show the evolution of burning regions and shells as a function of model number. This should look similar to the plot you made for the $5\,\mathrm{M}_\odot$ star with the exception of the final models, which show carbon ignition in the core.

Use an HR diagram to determine the timescale of each stellar evolution phase, as you did for the lower mass stars. Compare your results. Can you explain why the main sequence timescale is different? In section 3.3 you determined $L \propto M^n$. Estimate the main sequence lifetime of a star using this formula.

Plot the HR diagram and label it with *model number*. One feature you should see is the *blue loop* between models 900 and 2900. What phase of stellar evolution does this correspond to? How long does the blue loop last?

Does the star lose a significant amount of mass before the end of the evolution is reached? Why not? Compare this to the $5\,\mathrm{M}_\odot$ and discuss the effect this has on the burning phases which can be reached in a 5 and a $10\,\mathrm{M}_\odot$ star.

Plot the surface abundances of H, He, C, N and O as a function of time (or model number). In which phases of evolution do the major changes in surface abundance take place? (Hint: compare to your HRD) What is the cause of these changes? Compare to your $5\,\mathrm{M}_\odot$ star.

Plot central temperature against central density. Is the core of this star ever degenerate?

Examine the internal abundances in the final few models. What is the mass of the *helium core*, *CO core* and the *ONe core*. What is the composition of the ONe core? The stellar evolution code does not distinguish between the isotopes $^{20}\mathrm{Ne}$ and $^{22}\mathrm{Ne}$. Estimate the amount of neon in each isotope. The solar ratio of $^{20}\mathrm{Ne}/^{22}\mathrm{Ne}$ is about 12.4 – is the ONe core a likely source of the solar system material?

What is the highest temperature in the core in the final model? Is the central temperature ever different from the maximum temperature? Plot the central temperature as a function of age for the last 50, 000 years. Is the temperature increasing or decreasing, and is it speeding up or slowing down?

For each phase of nucleosynthesis there is a fixed amount of fuel available and once this runs out the core contracts until the next fuel ignites. Eventually it burns all the way to iron and then there is no more fuel and nothing to stop the contraction except electron degeneracy pressure. What is the mass limit for the core to be supported by electron degeneracy pressure?

Once the core has grown large enough that this limit is overcome it collapses again with an associated increase in temperature and density. As the temperature increases, so does the number density of energetic photons so that endothermic *photodisintigration* reactions can take place. The photodisintigration reaction $^{20}\mathrm{Ne}(\gamma,\alpha)^{16}\mathrm{O}$ costs about 0.9 MeV of energy. Estimate the temperature at which this becomes important. This is the first of many photodisintigration reactions which occur. What is the effect of photodisintigration on the pressure and the molecular weight?

What is the end point of photodisintigration assuming in stars? The star will then continue to collapse and the pressure becomes so high that protons and electrons merge (inverse beta decay). What are the resulting particles and what happens to them and why? What happens to the pressure when protons and electrons merge? What is the core of the star made of at this point and what prevents its further collapse? What happens to the rest of the star? What is the radius of the core compared to a white dwarf of the same mass? Why are white dwarfs stable against inverse beta decay? If a neutron star accretes material indefinitely, what is its fate and why?

When the neutron star is formed some of the material outside it (the *mantle* of the star) is lost in a supernova explosion. What elements do you expect to be present in the explosion ejecta? Could these explain the solar system abundances?

### 6.2 A $60\,\mathrm{M}_\odot$ star

Use your previous estimate for the main sequence timescale to predict the lifetime of a $60\,\mathrm{M}_\odot$ star. Evolve a $60\,\mathrm{M}_\odot$, $Z = 0.02$ star from the ZAMS. How does your estimate compare to the actual lifetime of the star?

Plot an HRD and identify the main sequence and subsequent evolutionary phases. In which direction does the star move in the HRD as it evolves?

Make a Kippenhahn plot of *E_nuc* vs mass and model number, as you did for the $10\,\mathrm{M}_\odot$ star. Comment on the structure of the burning phases. You should immediately notice one difference between this and the $10\,\mathrm{M}_\odot$ star: mass loss. The star loses mass *throughout its lifetime.* What is the mass-loss rate during the main sequence? What is the peak mass loss rate? What is the physical mechanism for the mass loss, and why does it happen in these stars but not in the $10\,\mathrm{M}_\odot$ star? Why is the peak mass loss rate where it is in the HRD and what type of star is it?

What is the effect on the surface abundances due to the mass loss? Stars with surface abundances of hydrogen less than 40% and effective temperatures above 10, 000 K are associated with peculiar stars called *Wolf-Rayet* (WR) stars. Does this star become a WR star?

Make a Kippenhahn plot of convection. Do you see the first and second dredge up?

Try evolving the same star without mass loss.

## 7 Composition: $Z = 10^{-4}$ vs $Z = 0.02$

Evolve a $5\,\mathrm{M}_\odot$ star as in section 5 but select $Z = 0.0001$. What are the initial abundances for a metallicity of $10^{-4}$?

Compare this model to your previous $Z = 0.02$, $5\,\mathrm{M}_\odot$ model.

- What is the luminosity at the ZAMS? What is the radius at the ZAMS? Are these different to $Z = 0.02$?
- In the HRD, can you locate the (first) giant branch?
- Make a Kippenhahn plot of convection vs model number and mass co-ordinate. Can you locate the first giant branch? If you cannot, try changing the $y$-axis to plot the *radius* instead. You should see a small increase around model 700, but it is clear that there is no true giant branch. Try to explain this in terms of the metallicity.
- Is there a first dredge up in this star? If not, why not?
- Why is there a loop in the HRD around model 1500?
- Is there a “second” dredge up as the star ascends the AGB? What is the mass of the hydrogen exhausted core before and after second dredge up?
- What are the surface abundance changes at “second” dredge up as a function of model number?
- What are the surface abundance changes as a function of time?
- Can you explain which nuclear reactions cause which change in the abundances?
- Between models 2100 and 2150 a phenomenon which is new to you occurs. What does it do to the surface abundances? Why does it happen?
- Plot the log temperature and log density vs mass coordinate for model 2140 in the mass range $1.00\,\mathrm{M}_\odot$ to $1.01\,\mathrm{M}_\odot$. By how many orders of magnitude do the temperature and density drop? Over the same mass range plot $\nabla_{\mathrm{rad}} - \nabla_{\mathrm{ad}}$. Remember that when this is positive there is convection and mixing. At what mass coordinate does the convection envelope start? (Hint: zoom in as necessary to give an accurate answer) What amount of mass is located in a region with $\log_{10} T/\mathrm{K} > 7.5$ *and* in the convective envelope? This might be a very small region, but remember that it is both hot and mixed continuously so there is no shortage of hydrogen. Hence burning is quite efficient and much of the envelope is CNO cycled. This process is thought to be one of the primary contributors to nitrogen in the universe.

- What stops hot bottom burning?

- During the AGB what is the composition of the core? Is this different from the $Z = 0.02$, $5\,\mathrm{M}_\odot$ star? What is the mass of the *hydrogen exhausted* core at the beginning of the AGB, and at the end of the evolution? How much has the core grown during the AGB?

- Mass loss eventually strips the envelope. What is left behind?